Publ. Math. Debrecen
In-print:: Ref. no.: 8332 (2018), 1–2

Erratum to the paper: "On the Diophantine equations $(x-1)^3+x^5+(x+1)^3=y^n$ and $(x-1)^5+x^3+(x+1)^5=y^n$"

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In the proof of [2, Theorem 1.2], we miss the case $5|x$ for n an odd prime. When $5\nmid x$, since $\gcd(2x^2+1,x)=1$, $\gcd(x^2+10,x)=1$ or 2, one has $x=2^\beta u^n$, $19^\alpha z^n=2x^2+1=2^{2\beta+1}u^{2n}+1$, and then Lemma 2.3 can be used to get the result. But when $5|x$, since $\gcd(x^2+10,x)=5$ or 10, one has $x=2^\beta\times 5^{n-1}u^n$, $19^\alpha z^n=2x^2+1=2^{2\beta+1}\times 5^{2n-2}u^{2n}+1$, and then the following lemma can be used to obtain the result.

Lemma 1. *The binomial Thue equation*

$$19^\alpha z^p-2^{2\beta+1}\times 5^{2p-2}y^{2p}=1$$

has no integer solutions with p an odd prime and $y\neq 0$, where $\alpha=0,1$ or $p-1$ and $\beta=0$ or $p-1$.

Proof. When $\alpha=0$, the lemma can be obtained from [1, Theorem 1.1].

When $\alpha=1$ or $p-1$, and $p\geq 7$, we can associate the Frey curves

$$E: Y^2=X(X+1)(X-2^{2\beta+1}\times 5^{2p-2}y^{2p}).$$

There is a newform of level $N(E)_p=190$ for $\beta=p-1$, or a newform of level $N(E)_p=3040$ for $\beta=0$ such that $E\sim_p f$. Then we can get $p\leq 5$. The case $p=3,5$ can be obtained by solving Thue equations using Magma. □

Mathematics Subject Classification: 11D41, 11D61.
Key words and phrases: Diophantine equations, modular form, Thue equations.

References

[1] M. A. Bennett, K. Győry, M. Mignotte and Á. Pintér, Binomial Thue equations and polynomial powers, *Compos. Math.* **142** (2006), 1103–1121.

[2] Z. Zhang, On the Diophantine equations $(x-1)^3+x^5+(x+1)^3=y^n$ and $(x-1)^5+x^3+(x+1)^5=y^n$, *Publ. Math. Debrecen* **91** (2017), 383–390.

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(Received May 4, 2018)