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# Montel's criterion and shared function

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**Abstract.** In this paper, we prove some normal criteria for families of meromorphic functions that concern sharing values or functions, which improve and generalize Montel's normality criterion and the related results of Chang–Fang–Zalcman, and Sun et al. Some examples are given to show that the sharpness of our results.

## 1. Introduction

Let $\mathcal{F}$ be a family of meromorphic functions defined in $D$. $\mathcal{F}$ is said to be normal in $D$, in the sense of Montel, if for any sequence $f_n \in \mathcal{F}$ there exists a subsequence $f_{n_j}$, such that $f_{n_j}$ converges spherically, locally and uniformly in $D$, to a meromorphic function or $\infty$(see [6], [12]).

The most celebrated theorem in the theory of normal families is the following criterion of Montel [6], who created the theory of normal families.

**Theorem A.** *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain $D$. If, for every function $f \in \mathcal{F}$, $f \neq 0, 1, \infty$ in $D$, then $\mathcal{F}$ is normal.*

In past years, this result has undergone various extensions (for details, see [1]–[5], [7]–[10] etc.).

Let $f$, $g$ and $\psi$ be three meromorphic functions in $D$. If $f(z) - \psi(z)$ and $g(z) - \psi(z)$ have the same zeros (ignoring multiplicity), we say that $f$ and $g$ share function $\psi$ in $D$. If $\psi$ is a constant $a$, then $f$ and $g$ share the value $a$ in $D$. Sun [8] proved the following general result.

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**Theorem B.** *Let $\mathcal{F}$ be a family of meromorphic functions defined in $D$. If, for each pair of functions $f$ and $g$ in $\mathcal{F}$, $f$ and $g$ share $0, 1, \infty$ in $D$, then $\mathcal{F}$ is normal.*

*Remark 1.* As we know, the classical Nevanlinna's five point theorem asserts that: if two non-constant meromorphic functions $f$ and $g$ share five distinct values in the complex plane, then $f \equiv g$. Here Theorem B only supposes that each pair of functions $f$ and $g$ in $\mathcal{F}$, $f$ and $g$ share three distinct values in a domain $D$ (not the whole plane), so the family $\mathcal{F}$ in Theorem B is not trivial.

It is natural to consider the case that each pair of functions $f, g \in \mathcal{F}$ share two value or one value. In this paper, we prove the following results.

**Theorem 1.** *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain $D \subset \mathbb{C}$. Suppose that* (1) *for each pair of functions $f, g \in \mathcal{F}$, $f$ and $g$ share $0, \infty$ in $D$;* (2) *all zeros of $f - 1$ are multiple for each $f \in \mathcal{F}$ in $D$. Then $\mathcal{F}$ is normal in $D$.*

**Theorem 2.** *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain $D \subset \mathbb{C}$. Suppose that* (1) *for each pair of functions $f, g \in \mathcal{F}$, $f$ and $g$ share $0$ in $D$;* (2) *all poles of $f$ have multiplicity at least $2$ (or $3$) and all zeros of $f - 1$ have multiplicity at least $3$ (or $2$) in $D$ for each $f \in \mathcal{F}$ in $D$. Then $\mathcal{F}$ is normal in $D$.*

*Remark 2.* If $f$ and $g$ share other two values (in Theorem 1), or other one value (in Theorem 2), similarly results can be proved by using the same argument as in this paper.

We further extend the constant '1' in Theorem 1, 2 and B to a general function '$\psi(z)$', as follows.

**Theorem 3.** *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain $D$, and let $\psi(z)$ be a meromorphic function such that $\psi(z) \not\equiv 0, \infty$ in $D$. Suppose that* (1) *for each pair of functions $f, g \in \mathcal{F}$, $f$ and $g$ share $0, \infty$ in $D$;* (2) *all zeros of $f - \psi$ are multiple in $D$ for $f \in \mathcal{F}$;* (3) *the multiplicity of $f \in \mathcal{F}$ is larger than that of $\psi(z)$ at the common zeros or poles of $f$ and $\psi(z)$ in $D$. Then $\mathcal{F}$ is normal in $D$.*

**Theorem 4.** *Let $\mathcal{F}$ be a family of meromorphic functions defined in a domain $D$, let $\psi(z)$ be a holomorphic function such that $\psi(z) \not\equiv 0$ in $D$. Suppose that* (1) *for each pair of functions $f, g \in \mathcal{F}$, $f$ and $g$ share $0$ in $D$;* (2) *all poles of $f$ have multiplicity at least $2$ (or $3$) and all zeros of $f - \psi$ have multiplicity at* *least* 3 *(or* 2*) for each* $f \in \mathcal{F}$ *in* $D$*;* (3) *the multiplicity of* $f \in \mathcal{F}$ *is larger than that of* $\psi(z)$ *at the common zeros of* $f$ *and* $\psi(z)$ *in* $D$*. Then* $\mathcal{F}$ *is normal in* $D$*.*

**Theorem 5.** *Let* $\mathcal{F}$ *be a family of meromorphic functions defined in a domain* $D$*, let* $\psi(z)$ *be a meromorphic function such that* $\psi(z) \not\equiv 0, \infty$ *in* $D$*. Suppose that* (1) *for each pair of functions* $f, g \in \mathcal{F}$*,* $f$ *and* $g$ *share* $0, \infty, \psi(z)$ *in* $D$*;* (2) *the multiplicity of* $f \in \mathcal{F}$ *is larger than that of* $\psi(z)$ *at the common zeros or poles of* $f$ *and* $\psi(z)$ *in* $D$*. Then* $\mathcal{F}$ *is normal in* $D$*.*

*Remark 3.* In [2], Chang, Fang and Zalcman proved that Montel's criterion is still valid if $0, 1, \infty$ are replaced by three distinct meromorphic functions $a(z), b(z), c(z)$. Clearly, our above theorems extend their result in some sense.

*Remark 4.* The condition that the multiplicity of $f \in \mathcal{F}$ is larger than that of $\psi(z)$ at the common zeros *or* poles of $f$ and $\psi(z)$ in $D$ in Theorem 3–5 cannot be omitted, as is shown by the following examples.

*Example 1.* Let $D = \{z : |z| < 1\}$, $\psi(z) = z^k$, where $k \geq 2$ is a positive integer, and

$$\mathcal{F} = \{f_n(z) = nz^2,\ n = 2, 3, \dots\}.$$

Clearly, for each pair of functions $f_n, f_m \in \mathcal{F}$, $f_n(z)$ and $f_m(z)$ share $0, \infty$ in $D$. Since $f_n(z) - \psi_(z) = (n - z^{k-2})z^2$ and $f_m(z) - \psi(z) = (m - z^{k-2})z^2$, $f_n(z)$ and $f_m(z)$ share $\psi(z)$ in $D$. But $\mathcal{F}$ is not normal in $D$.

*Example 2.* Let $D = \{z : |z| < 1\}$, $\psi(z) = 1/z^k$, where $k \geq 1$ is a positive integer, and

$$\mathcal{F} = \Big\{f_n(z) = \frac{1}{nz},\ n = 2, 3, \dots\Big\}.$$

Clearly, for each pair of functions $f_n, f_m \in \mathcal{F}$, $f_n(z)$ and $f_m(z)$ share $0, \infty$ in $D$. Since

$$f_n(z) - \psi(z) = \frac{z^{k-1} - n}{nz^k}, \qquad f_m(z) - \psi(z) = \frac{z^{k-1} - m}{mz^k},$$

$f_n(z) - \psi(z)$ and $f_m(z) - \psi(z)$ have no zeros in $D$, so that $f_n(z)$ and $f_m(z)$ share $\psi(z)$ in $D$. But $\mathcal{F}$ is not normal in $D$.

*Remark 5.* The above examples also show that $\psi(z) \not\equiv 0, \infty$ in Theorem 3–5 is necessary.

## 2. Lemmas

We first give some notations. $D_r(z_0)$ is the open disc of radius $r$ and center $z_0$, and $D'_r(z_0)$ is the same disc minus its center.

The following is the local version of Zalcman's lemma [11] (see also [12]).

**Lemma 1.** *Let $\mathcal{F}$ be a family of functions meromorphic in a domain $D$. If $\mathcal{F}$ is not normal at $z_0 \in D$, then there exist a sequence of points $z_n \in D$, $z_n \to z_0$, a sequence of positive numbers $\rho_n \to 0$, and a sequence of functions $f_n \in \mathcal{F}$ such that*

$$g_n(\zeta) = f_n(z_n + \rho_n \zeta) \to g(\zeta)$$

*locally uniformly with respect to the spherical metric, where $g$ is a nonconstant meromorphic function on $\mathbb{C}$.*

The next is a fundamental result in the normal theory, which has appeared in the proofs of some papers. For brevity, we here state it independently, and give its proof.

**Lemma 2.** *Let $\mathcal{F}$ be a family of functions holomorphic in $D_r(z_0)$. Suppose that $\mathcal{F}$ is normal in $D'_r(z_0)$, but not normal at $z_0$. Then there exists a sequence $\{f_n\} \subset \mathcal{F}$ such that $f_n \to \infty$ locally uniformly in $D'_r(z_0)$.*

Proof. Since $\mathcal{F}$ is normal in $D'_r(z_0)$, but not normal at $z_0$, there exists a sequence $\{f_n\} \subset \mathcal{F}$ such that $f_n$ converges to a function $h$ or the constant infinity locally uniformly in $D'_r(z_0)$, but not in $D_r(z_0)$.

If $f_n(z) \to h(z)$ in $D'_r(z_0)$, then $h$ is holomorphic in $D'_r(z_0)$ since $f_n$ is holomorphic in $D_r(z_0)$. For each $0 < r' < r$, there exists a positive number $M$ such that $|h(z)| \le M$ on $|z - z_0| = r'$. It follows that $|f_n(z)| \le 2M$ on $|z - z_0| = r'$ for large $n$. The maximum modulus theorem implies that $|f_n(z)| \le 2M$ holds in $\bar{D}_{r'}(z_0) = \{z : |z - z_0| \le r'\}$. Then $h$ is bounded in $\bar{D}_{r'}(z_0)$, and thus $h$ extends to be holomorphic in $\bar{D}_{r'}(z_0)$. Again by the maximum modulus theorem, $f_n(z) \to h(z)$ in $\bar{D}_{r'}(z_0)$, so that $f_n(z) \to h(z)$ in $D_r(z_0)$, a contradiction.

Thus $f_n \to \infty$ on compact subsets of $D'_r(z_0)$. Lemma 2 is proved. $\square$

## 3. Proof of Theorems

Proof of Theorem 1. Since normality is a local property, it is enough to show that $\mathcal{F}$ is normal at each $z_0 \in D$.

Suppose not; then, by Lemma 1, there exist functions $f_n \in \mathcal{F}$, points $z_n \to z_0$ and positive numbers $\rho_n \to 0$, such that

$$g_n(\zeta) = f_n(z_n + \rho_n\zeta) \to g(\zeta)$$

converges spherically uniformly on compact subsets of $\mathbb{C}$, where $g(\zeta)$ is a nonconstant meromorphic function on $\mathbb{C}$.

We divide into three cases.

*Case 1.* There exists $f \in \mathcal{F}$ such that $f(z_0) \neq 0, \infty$.

Then we can find $r > 0$ such that $D_r(z_0) \subset D$, and $f(z) \neq 0, \infty$ in $D_r(z_0)$. By the assumptions of Theorem 1, we know that these still hold for each $f \in \mathcal{F}$.

It follows that $g \neq 0$, and $g$ is entire in $\mathbb{C}$. Since

$$g_n(\zeta) - 1 = f_n(z_n + \rho_n\zeta) - 1 \to g(\zeta) - 1,$$

and $f_n - 1$ has only multiple zeros, $g - 1$ has no simple zeros. Nevanlinna's fundamental theorem implies that $g$ is a constant, a contradiction.

*Case 2* There exists $f \in \mathcal{F}$ such that $f(z_0) = 0$.

Then we may choose $r > 0$ such that $D_r(z_0) = \{z : |z - z_0| < r\} \subset D$, and $f(z) \neq 0$ in $D'_r(z_0)$ and $f(z) \neq \infty$ in $z \in D_r(z_0)$. Also by the assumptions of Theorem 1, these still hold for each $f \in \mathcal{F}$.

As in Case 1, $g$ is entire and $g - 1$ has only multiple zeros in $\mathbb{C}$.

Furthermore, we claim that $g$ has at most one zero point in $\mathbb{C}$. Indeed, note the fact that each $f_n$ has only one zero at $z_0$ in $D_r(z_0)$ and $g$ is nonconstant. If $(z_n - z_0)/\rho_n \to \infty$, in view of $g_n(-(z_n - z_0)/\rho_n) = f_n(z_0)$, then the zero of $g_n$ corresponding to that of $f_n$ at $z_0$ drifts off to infinity, and thus $g$ has no zeros. If $(z_n - z_0)/\rho_n \not\to \infty$, taking a subsequence and renumbering, we may assume that $(z_n - z_0)/\rho_n \to \alpha$ (a finite complex number). Since

$$f_n(\rho_n\zeta + z_0) = f_n\left(z_n + \rho_n\left(\zeta - \frac{z_n - z_0}{\rho_n}\right)\right) = g_n\left(\zeta - \frac{z_n - z_0}{\rho_n}\right) \to g(\zeta - \alpha),$$

we deduce that $g$ has only one zero at $-\alpha$.

Hence, by Nevanlinna's fundamental theorem, $g$ is not transcendental. Then $g$ has the form

$$g(\zeta) = A(\zeta - B)^l$$

where $A(\neq 0), B$ are constants, $l$ is a positive integer. So $g(\zeta) - 1 = A(\zeta - B)^l - 1$ has only simple zeros, a contradiction.

*Case 3.* There exists $f \in \mathcal{F}$ such that $f(z_0) = \infty$.

As before, we can find $r > 0$ such that $D_r(z_0) \subset D$, and $f(z) \neq 0$ in $D_r(z_0)$ and $f(z) \neq \infty$ in $D'_r(z_0)$ for each $f \in \mathcal{F}$.

Consider the family $\mathcal{F}_1 = \{1/f : f \in \mathcal{F}\}$. Then $\mathcal{F}_1$ is not normal at $z_0$. As Case 2, we can derive a contradiction. Theorem 1 is proved. □

Proof of Theorem 2. Suppose $\mathcal{F}$ is normal at $z_0 \in D$. Then, by Lemma 1, there exist functions $f_n \in \mathcal{F}$, points $z_n \to z_0$ and positive numbers $\rho_n \to 0$, such that

$$g_n(\zeta) = f_n(z_n + \rho_n\zeta) \to g(\zeta)$$

converges spherically uniformly on compact subsets of $\mathbb{C}$, where $g(\zeta)$ is a non-constant meromorphic function on $\mathbb{C}$.

Now we divide into two cases.

*Case 1.* There exists $f \in \mathcal{F}$ such that $f(z_0) \neq 0$.

Then we can find $r > 0$ such that $D_r(z_0) \subset D$, and $f(z) \neq 0$ for $z \in D_r(z_0)$. By the assumptions of Theorem 2, we know that this still holds for each $f \in \mathcal{F}$.

Hurwitz's theorem implies that that $g \neq 0$. Since all poles of $f_n$ have multiplicity at least 2 (or 3) and all zeros of $f_n - 1$ have multiplicity at least 3 (or 2), all poles of $g$ have multiplicity at least 2(or 3) and all zeros of $g - 1$ have multiplicity at least 3 (or 2). By Nevanlinna's fundamental theorem, $g$ is a constant, a contradiction.

*Case 2.* There exists $f \in \mathcal{F}$ such that $f(z_0) = 0$.

As before, we can find $r > 0$ such that $D_r(z_0) \subset D$, and $f(z) \neq 0$ in $D'_r(z_0)$ and $f(z) \neq \infty$ in $z \in D_r(z_0)$ for each $f \in \mathcal{F}$.

Then, $g$ is entire in $\mathbb{C}$ and all zeros of $g-1$ have multiplicity at least 3 (or 2). Using the same argument as in Case 2 of the proof of Theorem 1, we conclude that $g$ has at most one zero, and then we can derive a contradiction. Theorem 2 is thus proved. □

Proof of Theorem 3. Let $z_0 \in D$. We distinguish the following three cases.

*Case 1.* $\psi(z_0) \neq 0, \infty$.

Arguing similarly as the proof of Theorem 1, we can prove that $\mathcal{F}$ is normal at $z_0$ for this case.

*Case 2.* $\psi(z_0) = 0$.

Then there exists $r > 0$ such that $D_r(z_0) \subset D$ and $\psi(z) \neq \infty$ in $D_r(z_0)$, and $\psi(z) \neq 0$ in $D'_r(z_0)$. By Case 1, $\mathcal{F}$ is normal in $D'_r(z_0)$. Set

$$\mathcal{G} = \left\{ F = \frac{f}{\psi} : f \in \mathcal{F} \right\}. \tag{$*$}$$

Since for each pair of functions $f, g \in \mathcal{F}$, $f$ and $g$ share $0, \infty$ in $D$ and all zeros of $f - \psi$ are multiple for $f \in \mathcal{F}$, noting that the multiplicity of $f \in \mathcal{F}$ is larger than that of $\psi(z)$ at the common zeros or poles of $f$ and $\psi(z)$ in $D$, we deduce that, for each pair of functions $F, G \in \mathcal{G}$, $F$ and $G$ share $0, \infty$ in $D$, and all zeros of $F - 1$ in $D$ are multiple for $F \in \mathcal{G}$. So, by Theorem 1, $\mathcal{G}$ is normal in $D$.

Now we distinguish two subcases.

*Case 2.1.* There exists $f \in \mathcal{F}$ such that $f(z_0) \neq 0$.

Then we can find $0 < r_1 < r$ such that $f \neq 0$ in $D_{r_1}(z_0)$. It follows from the assumptions of Theorem 3 that $f \neq 0$ in $D_{r_1}(z_0)$ for each $f \in \mathcal{F}$. Thus, for all $F \in \mathcal{G}$, $F(z_0) = f(z_0)/\psi(z_0) = \infty$.

Since $\mathcal{G}$ is normal at $z_0$, $\mathcal{G}$ is equicontinuous at $z_0$ with respect to the spherical distance. Hence, there exists $0 < r_2 < r_1$ such that $|F(z)| \geq 1$ for all $F \in \mathcal{G}$ and $z \in D_{r_2}(z_0)$. On the other hand, $\mathcal{F}$ is normal in $D'_{r_2}(z_0)$, but not normal at $z_0$, then the family $\mathcal{F}_1 = \{1/f : f \in \mathcal{F}\}$ is normal in $D'_{r_2}(z_0)$, but not normal at $z_0$. Note that $\mathcal{F}_1$ is holomorphic in $D_{r_2}(z_0)$, by Lemma 2, there exists a sequence $\{1/f_n\} \subset \mathcal{F}_1$ such that $1/f_n \to \infty$ in $D'_{r_2}(z_0)$. Then $f_n \to 0$ converges locally uniformly in $D'_{r_2}(z_0)$, and hence so does $\{F_n\} \subset \mathcal{G}$, where $F_n = f_n/\psi$. But $|F_n(z)| \geq 1$ for each $z \in D_{r_2}(z_0)$, we derive a contradiction.

*Case 2.2.* There exists $f \in \mathcal{F}$ such that $f(z_0) = 0$.

As before, we can find $0 < r_1 < r$ such that $f$ is holomorphic in $D_{r_1}(z_0)$ and $f(z_0) = 0$ for each $f \in \mathcal{F}$.

Since the multiplicity of $f \in \mathcal{F}$ is larger than that of $\psi(z)$ at the common zeros of $f$ and $\psi(z)$, we have that $F(z_0) = f(z_0)/\psi(z_0) = 0$ for each $F \in \mathcal{G}$. As above, there exists $0 < r_2 < r_1$ such that $|F(z)| \leq 1$ for all $F \in \mathcal{G}$ and $z \in D_{r_2}(z_0)$. On the other hand, $\mathcal{F}$ is normal in $D'_{r_2}(z_0)$, but not normal at $z_0$. Since $f_n$ is holomorphic in $D_{r_2}(z_0)$ for each $n$, again by Lemma 2, $f_n \to \infty$ in $D'_{r_2}(z_0)$. It follows that $F_n \to \infty$ in $D'_{r_2}(z_0)$, where $F_n = f_n/\psi$. But $|F_n(z)| \leq 1$ for each $z \in D_{r_2}(z_0)$, a contradiction.

*Case 3.* $\psi(z_0) = \infty$.

There exists $r > 0$ such that $D_r(z_0) \subset D$, and $\psi(z) \neq 0$ in $D_r(z_0)$ and $\psi(z) \neq \infty$ in $D'_r(z_0)$. Then, by Case 1, $\mathcal{F}$ is normal in $D'_r(z_0)$.

Consider the family $\mathcal{G}$ defined as in $(*)$. Similarly as above, $\mathcal{G}$ is normal in $D$. we also distinguish two subcases.

*Case 3.1.* There exists $f \in \mathcal{F}$ such that $f(z_0) \neq \infty$.

Then we can choose $0 < r_1 < r$ such that $f \neq \infty$ in $D_{r_1}(z_0)$, and this also holds for each $f \in \mathcal{F}$ by the assumptions of Theorem 3. Thus, for all $F \in \mathcal{G}$, $F(z_0) = f(z_0)/\psi(z_0) = 0$. We can derive a contradiction as Case 2.2.

*Case 3.2.* There exists $f \in \mathcal{F}$ such that $f(z_0) = \infty$.

As above, we can find $0 < r_1 < r$ such that $f(z_0) = \infty$ and $f \neq 0$ in $D_{r_1}(z_0)$ for each $f \in \mathcal{F}$. Since the multiplicity of $f \in \mathcal{F}$ is larger than that of $\psi(z)$ at the common poles of $f$ and $\psi(z)$, we know that $F(z_0) = f(z_0)/\psi(z_0) = \infty$ for all $F \in \mathcal{G}$. Similarly as in Case 2.1, we can obtain a contradiction.

Theorem 3 is thus proved. □

PROOF OF THEOREM 4 AND THEOREM 5. Theorem 4 and Theorem 5 can be proved by using almost the same argument as in Theorem 3. We here omit the details. □

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