

Year: 2010

Vol.: 77

Fasc.: 1-2

**Title:** On the distribution mod 1 of  $\alpha\sigma(n)$

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The sequence  $x_n = F(n) + \alpha\sigma(n) \pmod{1}$  is investigated, where  $\sigma(n)$  = sum of divisors of  $n$ ,  $F$  is an additive arithmetical function. In an earlier paper De Koninck and the author proved that  $x_n \pmod{1}$  is uniformly distributed if the approximation type of  $\alpha$  is finite, and formulated the conjecture that it holds for every irrational  $\alpha$ . In this paper it is proved that the conjecture is not true in general, and it is true if  $\alpha \in \mathcal{K}^*$ .  $\mathcal{K}^*$  is defined as follows. Let  $M_x = \prod_p p^{r_p}$ ,  $p$  runs over the primes and  $r_p$  is the integer part of the number stated in the right hand side of (2.7). Let  $\mathcal{K} = \mathcal{K}_x$  be the set of those irrational  $\alpha$ , for which  $\min_{H|M_x} \|H\alpha\|x > 1$  holds for every large  $x$ ,  $\mathcal{K}^* = \{\alpha \mid j\alpha \in \mathcal{K}\}$  for every  $j = 1, 2, \dots$ .

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