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Title: On the exponential diophantine equation $(a^n - 1)(b^n - 1) = x^2$

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Let a and b be fixed positive integers such that $a \neq b$ and $\min(a, b) > 1$. In this paper, we combine some divisibility properties of the solutions of Pell equations with elementary arguments to prove that if $a \equiv 2 \pmod{6}$ and $b \equiv 0 \pmod{3}$, then the title equation $(a^n - 1)(b^n - 1) = x^2$ has no positive integer solution (n, x) . Moreover, we show that in case of $a \equiv 2 \pmod{20}$ and $b \equiv 5 \pmod{20}$, where $b - 1$ is a full square, the only possible solution belongs to $n = 1$.

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