Let $x : M^n \to \mathcal{M}_1^{n+1}(c)$ be a complete spacelike hypersurface immersed into a Lorentzian space form, where $\mathcal{M}_1^{n+1}(c)$ is a Lorentz–Minkowski space $\mathbb{L}^{n+1} = \mathbb{R}^{n+1}$, a de Sitter space $\mathbb{S}^{n+1}_1 \subset \mathbb{R}^{n+2}_1$ or an anti-de Sitter space $\mathbb{H}^{n+1}_1 \subset \mathbb{R}^{n+2}_2$, according to $c = 0$, $c = 1$ or $c = -1$, respectively. Let $\phi = \langle x, a \rangle$ and $\psi = \langle \mathbf{H}, a \rangle$, where $\mathbf{H}$ is the mean curvature vector field of $M^n$ and $a$ is a fixed nonzero vector in the corresponding pseudo-Euclidean space. We prove that if $M^n$ has constant mean curvature (CMC), and $\phi = \lambda \psi$, for some real number $\lambda$, then $M^n$ is a spacelike isoparametric hypersurface of $\mathcal{M}_1^{n+1}(c)$. Furthermore, it is either a totally umbilical hypersurface or a hyperbolic cylinder.