Title: Functional equation of Dhombres type in the real case

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We consider continuous solutions \( f : \mathbb{R}_+ \to \mathbb{R}_+ = (0, \infty) \) of the functional equation \( f(xf(x)) = \varphi(f(x)) \) where \( \varphi \) is a given continuous map \( \mathbb{R}_+ \to \mathbb{R}_+ \). If \( \varphi \) is an increasing homeomorphism the solutions are completely described, if not there are only partial results. In this paper we bring some necessary conditions upon a possible range \( R_f \). In particular, if \( \varphi|_{R_f} \) has no periodic points except for fixed points then there are at most two fixed points in \( R_f \), and all possible types of \( R_f \) and all possible types of behavior of \( f \) can be described. The paper contains techniques which essentially simplify the description of the class of all solutions.

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