Let \( q \) be an odd positive integer and \( P \in F_2[z] \) be of order \( q \) and such that \( P(0) = 1 \). We denote by \( \mathcal{A} = \mathcal{A}(P) \) the unique set of positive integers satisfying \( \sum_{n=0}^{\infty} p(\mathcal{A}, n) z^n \equiv P(z) \pmod{2} \), where \( p(\mathcal{A}, n) \) is the number of partitions of \( n \) with parts in \( \mathcal{A} \). In [?], it is proved that if \( A(P, x) \) is the counting function of the set \( \mathcal{A}(P) \) then \( A(P, x) \ll x (\log x)^{-r/\varphi(q)} \), where \( r \) is the order of 2 modulo \( q \) and \( \varphi \) is the Euler’s function. In this paper, we improve on the constant \( c = c(q) \) for which \( A(P, x) \ll x (\log x)^{-c} \).