Title: A characterization of Beckenbach families admitting discontinuous Jensen affine functions

Author(s): Michal Lewicki

Let $F : \mathbb{R}^3 \to \mathbb{R}$ be a continuous function such that $F := \{ \mathbb{R} \ni x \mapsto F(x, a, b) \in \mathbb{R} : a, b \in \mathbb{R} \}$ is a Beckenbach family. Additionally, we assume that for each $a, b \in \mathbb{R}$ the functions $\mathbb{R} \ni x \mapsto F(x, a, b) \in \mathbb{R}$ are monotonic. We show that if there exists a function which is discontinuous at some point and Jensen affine with respect to the family $F$, then there exists a strictly increasing and continuous function $h : \mathbb{R} \to \mathbb{R}$ and continuous $G, H : \mathbb{R}^2 \to \mathbb{R}$ such that

$$F(u, a, b) = h(G(a, b)u + H(a, b)),$$

for all $u, a, b \in \mathbb{R}$. As a consequence we get an independent proof of theorem of J. Matkowski. Finally, we characterize Beckenbach families of the form ($\ast$).

Address:
Michal Lewicki
Institute of Mathematics
Silesian University
Bankowa 14
40–007 Katowice
Poland