Title: On the mod $p^2$ determination of $\sum_{k=1}^{p-1} H_k/(k \cdot 2^k)$: another proof of a conjecture by Sun

Author(s): Romeo Meštrović

For a positive integer $n$ let $H_n = \sum_{k=1}^{n} 1/k$ be the $n$th harmonic number. Z. W. Sun conjectured that for any prime $p \geq 5$,

$$\sum_{k=1}^{p-1} \frac{H_k}{k \cdot 2^k} \equiv \frac{7}{24} pB_{p-3} \pmod{p^2}.$$

This conjecture is recently confirmed by Z. W. Sun and L. L. Zhao. In this note we give another proof of the above congruence by establishing congruences for all the sums of the form $\sum_{k=1}^{p-1} 2^{\pm k} H_k^r / k^s \pmod{p^{4-r-s}}$ with $(r, s) \in \{(1, 1), (1, 2), (2, 1)\}$.

Address:
Romeo Meštrović
Department of Mathematics
Maritime Faculty
University of Montenegro
Dobrota 36
85330 Kotor
Montenegro