**Title:** Practical pretenders

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Following Srinivasan, an integer $n \geq 1$ is called *practical* if every natural number in $[1, n]$ can be written as a sum of distinct divisors of $n$. This motivates us to define $f(n)$ as the largest integer with the property that all of $1, 2, 3, \ldots, f(n)$ can be written as a sum of distinct divisors of $n$. (Thus, $n$ is practical precisely when $f(n) \geq n$.) We think of $f(n)$ as measuring the “practicality” of $n$; large values of $f$ correspond to numbers $n$ which we term *practical pretenders*. Our first theorem describes the distribution of these impostors: Uniformly for $4 \leq y \leq x$,

$$
\#\{n \leq x : f(n) \geq y\} \asymp \frac{x}{\log y}.
$$

This generalizes Saias’s result that the count of practical numbers in $[1, x]$ is $\asymp \frac{x}{\log x}$.

Next, we investigate the maximal order of $f$ when restricted to non-practical inputs. Strengthening a theorem of Hausman and Shapiro, we show that every $n > 3$ for which $f(n) \geq \sqrt{\gamma n \log \log n}$ is a practical number.

Finally, we study the range of $f$. Call a number $m$ belonging to the range of $f$ an *additive endpoint*. We show that for each fixed $\Lambda > 0$ and $\epsilon > 0$, the number of additive endpoints in $[1, x]$ is eventually smaller than $x/(\log x)^\Lambda$ but larger than $x^{1-\epsilon}$.

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