Title: A common structure of $n_k$'s for which $n_k\alpha \mod 1 \to x$

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Let $\alpha$ be an irrational number and $\varepsilon_k \leq 1, k = 1, 2, \ldots$, be an arbitrary decreasing sequence of real numbers such that $\varepsilon_k \to 0$. In this paper we show a construction of sequences $n_k, k = 1, 2, \ldots$, for which the fractional parts $\{n_k\alpha\} \to x$, where $x \in [0, 1]$ is fixed but arbitrary and $k/n_k \geq \varepsilon_k$ for $k = 1, 2, \ldots$. Here $\{n_k\alpha\} \in I_j$ for $k_{j-1} < k \leq k_j$ and the length $|I_j| = \{h_j\alpha\}$, where $h_j$ is a positive integer for $j = 1, 2, \ldots$. The increasing sequence $k_j$ is independent of $x$. Moreover, the differences $n_{k+1} - n_k$ satisfy the three gaps property with parameters $a_j, b_j$ and $a_j + b_j$ not depending on $x$ for every $k_{j-1} < k < k_j$ and $j = 2, 3, \ldots$.

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