Title: Rationality of the zeta function of the subgroups of abelian $p$-groups

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Given a finite abelian $p$-group $F$, we prove an efficient recursive formula for $\sigma_o(F) = \sum_{H \leq F} |H|^a$ where $H$ ranges over the subgroups of $F$. We infer from this formula that the $p$-component of the corresponding zeta-function on groups of $p$-rank bounded by some constant $r$ is rational with a simple denominator. We also provide two explicit examples in rank $r = 3$ and $r = 4$, as well as, a closed formula for $\sigma_o(F)$.

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