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Title: Non-Galois cubic number fields with exceptional units

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We consider the family of non-normal totally real cubic number fields $\mathbb{K}_l$ associated with the $\mathbb{Q}$-irreducible cubic polynomials $f_l(X) = X^3+(l-1)X^2-lX-1 \in \mathbb{Z}[X]$, $l \geq 3$. Let ε_l be a root of $f_l(X)$. Then ε_l and $\varepsilon_l - 1$ are units of $\mathbb{K}_l$. Let j_l denote the index of the groups of units generated by -1, ε_l and $\varepsilon_l - 1$ in the group of units $\mathbb{U}_l$ of the ring of algebraic integers of $\mathbb{K}_l$. V. Ennola proved in 1991 (i) that $\gcd(j_l, 2 \cdot 3 \cdot 5) = 1$ for $l \geq 3$, (ii) that $j_l = 1$ for $3 \leq l \leq 500$, and (iii) he conjectured that $j_l = 1$ for $l \geq 3$. We prove (i) that $\gcd(j_l, 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19) = 1$ for $l \geq 3$, and (ii) that $j_l = 1$ for $3 \leq l \leq 5 \cdot 10^7$, thus adding a lot more credit to this conjecture.

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