Title: Generalized symmetric \( * \)-rings and Jacobson’s Lemma for Moore–Penrose inverse

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It is well known as Jacobson’s Lemma that \( 1 - ba \) is invertible in a ring if so is \( 1 - ab \). Moreover, if \( c = (1 - ab)^{-1} \), then \( (1 - ba)^{-1} = 1 + bca \). However, the analogous statement for Moore–Penrose inverse in a \( * \)-ring is not true in general. Note that Jacobson’s Lemma for Moore–Penrose inverse holds true in a symmetric \( * \)-ring. In this paper, we study symmetric \( * \)-rings and introduce the notion of a generalized symmetric \( * \)-ring. A \( * \)-ring \( R \) is called generalized symmetric if \( 1 - (a^* - a)^2 \) is invertible for all units \( a \) in \( R \). When \( 1 - ab \) is Moore–Penrose invertible in such a ring, we provide sufficient and necessary conditions under which \( 1 - ba \) has a Moore–Penrose inverse \( (1 - ba)^\dagger \) and give a formula for \( (1 - ba)^\dagger \).

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