Local characterization of Jordan *-derivations on $B(H)$

Let $H$ be an infinite-dimensional real Hilbert space, and $B(H)$ the algebra of all bounded linear operators on $H$. Assume that $\delta : B(H) \to B(H)$ is a real linear map and $P \in B(H)$ is zero, or the unit element, or a nontrivial idempotent with infinite-dimensional range and infinite-dimensional kernel. It is shown that $\delta$ satisfies $\delta(A^2) = \delta(A)A^* + A\delta(A)$ for all $A \in B(H)$ with $A^2 = P$ if and only if $\delta$ is an inner Jordan *-derivation. An example is also given to illustrate that this is not necessarily true when $H$ is finite-dimensional.

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