

Title: On two open problems of the theory of permutable subgroups of finite groups

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Let $\sigma = \{\sigma_i | i \in I\}$ be some partition of the set of all primes \mathbb{P} , G a finite group and $\sigma(G) = \{\sigma_i | \sigma_i \cap \pi(G) \neq \emptyset\}$.

A set \mathcal{H} of subgroups of G is said to be a *complete Hall σ -set* of G if every member $\neq 1$ of \mathcal{H} is a Hall σ_i -subgroup of G for some $\sigma_i \in \sigma$ and \mathcal{H} contains exactly one Hall σ_i -subgroup of G for every $\sigma_i \in \sigma(G)$; G is said to be *σ -full* if G possesses a complete Hall σ -set.

A subgroup A of G is said to be *σ -permutable* in G if G possesses a complete Hall σ -set and A permutes with each Hall σ_i -subgroup H of G , that is, $AH = HA$ for all $i \in I$.

We prove that if G is σ -full, then the set $\mathcal{L}_{\sigma \text{ per}}(G)$, of all σ -permutable subgroups of G , forms a sublattice of the lattice of all subgroups of G . Also, answering to [9, Question 6.13], we describe the conditions under which the lattice $\mathcal{L}_{\sigma \text{ per}}(G)$ is distributive.

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