Title: Mean invariance identity

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For a continuous and increasing function \( f \) in a real interval \( I \), and a bivariable mean \( P \) defined in \( I^2 \), we prescribe a pair of bivariable means \( M \) and \( N \) such that the quasiarithmetic mean \( A_f \) generated by \( f \) is invariant with respect to the mean-type mapping \( (M, N) \). This allows to find effectively the limit of the iterates of the mean-type mapping \( (M, N) \). The means \( M \) and \( N \) are equal iff \( P \) is the arithmetic mean \( A \); they are symmetric iff so is \( P \). Treating \( f \) and \( P \) as the parameters, we obtain the family of all pairs of means \( (M, N) \) such that the quasiarithmetic mean \( A_f \) is invariant with respect to \( (M, N) \). In particular, we indicate the function \( f \) and the mean \( P \) such that the invariance identity \( A_f \circ (M, N) = A_f \) coincides with the equality \( G \circ (H, A) \), where \( G \) and \( H \) are the geometric and harmonic means, equivalent to the classical Pythagorean harmony proportion.

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