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**Title:** Rings in which every element is the sum of a left zero-divisor and an idempotent

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A ring  $R$  is called *left zero-clean* if every element is the sum of a left zero-divisor and an idempotent. This class of rings is a natural generalization of  $O$ -rings and nil-clean rings. We determine when a skew polynomial ring is a left zero-clean ring. It is proved that a ring  $R$  is left zero-clean if and only if the upper triangular matrix ring  $\mathbb{T}_n(R)$  is left zero-clean. It is shown that a commutative ring  $R$  is zero-clean if and only if the matrix ring  $\mathbb{M}_n(R)$  is zero-clean for every positive integer  $n \geq 1$ . We characterize the zero-clean matrix rings over fields. We also determine when a  $2 \times 2$  matrix  $A$  over a field is left zero-clean. A ring is called *uniquely left zero-clean* if every element is uniquely the sum of a left zero-divisor and an idempotent. We completely determine when a ring is uniquely left zero-clean.

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