Title: Interiors of continuous images of self-similar sets with overlaps

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Let $K$ be the attractor of the following iterated function system
\[ \{ S_1(x) = \lambda x, S_2(x) = \lambda x + c - \lambda, S_3(x) = \lambda x + 1 - \lambda \}, \]
where $S_1(I) \cap S_2(I) \neq \emptyset, (S_1(I) \cup S_2(I)) \cap S_3(I) = \emptyset$, and $I = [0, 1]$ is the convex hull of $K$. Let $d_1 = \frac{1-c-\lambda}{\lambda} < \frac{1}{1-c-\lambda} = d_2$. Suppose that $f$ is a continuous function defined on an open set $U \subset \mathbb{R}^2$. Denote the image
\[ f_U(K, K) = \{ f(x, y) : (x, y) \in (K \times K) \cap U \}. \]
If $\partial_x f$, $\partial_y f$ are continuous on $U$, and there is a point $(x_0, y_0) \in (K \times K) \cap U$ such that
\[ \frac{\partial_y f(x_0, y_0)}{\partial_x f(x_0, y_0)} \in (d_1, d_2) \quad \text{or} \quad \frac{\partial_x f(x_0, y_0)}{\partial_y f(x_0, y_0)} \in (d_1, d_2), \]
then $f_U(K, K)$ contains an interval. As a result, we let $c = \lambda = \frac{1}{3}$, and if
\[ f(x, y) = x^\alpha y^\beta (\alpha \beta \neq 0), \quad x^\alpha \pm y^\alpha (\alpha \neq 0), \quad \sin(x) \cos(y), \quad \text{or} \quad x \sin(xy), \]
then $f_U(C, C)$ contains an interval, where $C$ is the middle-third Cantor set.

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