Year: 2020 | **Vol.: 96** | **Fasc.: 3-4**

Title: Congruences involving binomial coefficients and Apéry-like numbers

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For $n = 0, 1, 2, \ldots$, let $W_n = \sum_{k=0}^{[n/3]} \binom{2k}{k}\binom{3k}{k}\binom{n}{3k}(-3)^{n-3k}$, where $[x]$ is the greatest integer not exceeding x. Then $\{W_n\}$ is an Apéry-like sequence. In this paper we deduce many congruences involving $\{W_n\}$, in particular, we determine $\sum_{k=0}^{p-1}\binom{2k}{k}\frac{W_k}{m^k} \pmod p$ for $m = -640332, -5292, -972, -108, -44, -27, -12, 8, 54, 243$ by using binary quadratic forms, where $p > 3$ is a prime. We also prove several congruences for generalized Apéry-like numbers, and pose 29 challenging conjectures on congruences involving binomial coefficients and Apéry-like numbers.

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