Title: On $k$-antichains in the unit $n$-cube

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A chain in the unit $n$-cube is a set $C \subset [0, 1]^n$ such that for every $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in $C$, we either have $x_i \leq y_i$ for all $i \in [n]$, or $x_i \geq y_i$ for all $i \in [n]$. We consider subsets $A$, of the unit $n$-cube $[0, 1]^n$, that satisfy

$$\text{card}(A \cap C) \leq k, \quad \text{for all chains } C \subset [0, 1]^n,$$

where $k$ is a fixed positive integer. We refer to such a set $A$ as a $k$-antichain. We show that the $(n - 1)$-dimensional Hausdorff measure of a $k$-antichain in $[0, 1]^n$ is at most $kn$ and that the bound is asymptotically sharp. Moreover, we conjecture that there exist $k$-antichains in $[0, 1]^n$ whose $(n - 1)$-dimensional Hausdorff measure equals $kn$, and we verify the validity of this conjecture when $n = 2$.

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