Year: 2021 | Vol.: 98 | Fasc.: 3-4

Title: Three supercongruences for Apéry numbers and Franel numbers

Author(s): Yong Zhang

The Apéry numbers A_n and the Franel numbers f_n are defined by

$$A_n = \sum_{k=0}^{n} \binom{n+k}{2k}^2 \binom{2k}{k}^2 \quad \text{and} \quad f_n = \sum_{k=0}^{n} \binom{n}{k}^3 (n = 0, 1, \dots,).$$

In this paper, we prove three supercongruences for Apéry numbers and Franel numbers conjectured by Z.-W. Sun. For any prime $p \geq 5$,

$$\frac{1}{n}\left(\sum_{k=0}^{pn-1}(2k+1)A_k - p\sum_{k=0}^{n-1}(2k+1)A_k\right) \equiv 0 \pmod{p^{4+3\nu_p(n)}},$$

$$\frac{1}{n^3}\left(\sum_{k=0}^{pn-1}(2k+1)^3A_k - p^3\sum_{k=0}^{n-1}(2k+1)^3A_k\right) \equiv 0 \pmod{p^{6+3\nu_p(n)}},$$

and, for any prime p,

$$\frac{1}{n^3}\left(\sum_{k=0}^{pn-1}(3k+2)(-1)^k f_k - p^2\sum_{k=0}^{n-1}(3k+2)(-1)^k f_k\right) \equiv 0 \pmod{p^3},$$

where $\nu_p(n)$ denotes the p-adic order of n.

Address:
Yong Zhang
Department of Mathematics and Physics
Nanjing Institute of Technology
Nanjing 211167
P. R. China