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Orientation-reversing actions on a solid torus which extend to a lens space

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Abstract. We consider all the finite orientation-reversing actions on a solid torus V_1 , write explicit representations for each equivalence class of action, and identify the quotient space. For fixed positive integers n and s, these groups divide into seven families $(1 \leq i \leq 7)$ with two groups in each family $G_{(i,j)}(n,s)$ and $G_{(i,j)}(n,s')$, where s'=s or 2s depending on certain conditions on s or n. The integer j relates to the quotient type (Bj,n). We consider the question of extending these actions to a lens space $L(p,q)=V_1\cup_{\alpha}V_2$. We show $G_{(i,j)}(n,s)$ extends to the 3-sphere \mathbb{S}^3 but not to L(2,1), and $G_{(i,j)}(n,s')$ extends to L(2,1) but not to \mathbb{S}^3 . All the non-orientable orbifolds having an Euler number zero Heegaard decomposition with finite fundamental group appear as quotient spaces of these actions. In addition, no orientation-reversing action on V_1 extends to L(p,q) for p>2.

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