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## Quotients of some identifying subcategories of the topological category

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**Abstract.** In this paper, let **K** denote an identifying subcategory of the category of all topological spaces such that the front adjunction mapping is a quotient mapping for any topological space. The objects in **K** are called **K**-spaces. Note that **K** is reflective and  $\mathbf{Top}_i$  (i=0,1 and 2) is a special case for **K**. For a **K**-space X, we give sufficient and necessary conditions for X/R to be a **K**-space. Meanwhile, we get the Isomorphism Theorem for **K**-spaces, with a particular emphasis on  $T_i$  spaces. For a  $T_i$  space  $(X,\tau)$ , we obtain the sufficient and necessary conditions for X/R to be  $T_i$  only in terms of the given equivalence relation and the space itself. Especially, when X is  $T_2$ , we find that X/R is  $T_2$  if and only if  $R \subseteq (X,\tau_R) \times (X,\tau_R)$  is closed. Moreover, we obtain the congruence relation of posets from the topological perspective.

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