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On the generalized Ramanujan–Nagell equation $D_1x^2 - D_2 = p^n$

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Abstract. Let D be a nonsquare positive integer, and let D_1, D_2 be positive integers such that $D_1D_2 = D$ and $\gcd(D_1, D_2) = 1$. Further, let p be an odd prime with $p \nmid D$. Denote by $N(D_1, D_2, p)$ the number of positive integer solutions (x, n) to the generalized Ramanujan–Nagell equation $D_1x^2 - D_2 = p^n$. In this paper, we prove $N(D_1, D_2, p) \leq 4$. Moreover, we generalize the case where $D_1 = 1$ to give the *exceptional* triple (D_1, D_2, p), which satisfies

$$p^\nu = D_1a^2 + \eta, \quad D_2 = \frac{1}{D_1}\left(\frac{p^m - \eta}{2a}\right)^2 - p^m, \qquad a, \nu, m \in \mathbb{N}, \quad \nu < m, \quad \eta \in \{1, -1\}.$$

In this case, it holds $N(D_1, D_2, p) \geq 3$. We prove that if (D_1, D_2, p) is non-exceptional, and the squarefree part of D_1 is at most 1001, then $N(D_1, D_2, p) \leq 3$.

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