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A lower bound for restricted exponential sums

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Abstract. Let $M(f;r) = \max_{|z|=r} |f(z)|$. Define $\mathcal{F}_n$, $\mathcal{G}_n$ to be the classes of exponential sums of the form $\sum_{k=1}^{n} \lambda_k e^{\lambda_k z}$ and $\sum_{k=1}^{n} e^{\lambda_k z}$, respectively, with $|\lambda_1| = \cdots = |\lambda_n| = 1$. For $r \in \left(0, \frac{1}{2}\right)$, we prove that $\inf_{f\in\mathcal{F}_n} M(f;r) \asymp nr^{n-1}/(n-1)!$, and establish the Turán–Govil type bound $\inf_{g\in\mathcal{G}_n} M(g';r)/M(g;r) \asymp r^{n-1}/(n-1)!$. Approximations of entire functions of exponential type $\sigma \leq 1$ on compact sets $K \subset \mathbb{C}$ by sums $f_n \in \mathcal{F}_n$, as well as representations of harmonics of a trigonometric polynomial $T_n(t)$ in the form of sums of its translations, $T_n(t-t_k)$, are also considered. In particular, we obtain a new Fejér type estimate for the leading harmonic $\tau_{2n}(t)$ of nonnegative polynomials $T_{2n}(t)$ of even degree $2n$.

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