

## D’Alembert’s other functional equation on monoids revisited

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**Abstract.** This paper extends previous results about the functional equation  $f(xy) - f(\sigma(y)x) = g(x)h(y)$  for unknown functions  $f, g, h$  on a monoid, where  $\sigma$  is an automorphic involution. In a previous joint paper with H. Stetkær, we found the solutions of this functional equation on monoids generated by their squares and regular monoids. Here we find that the solutions are the same on a larger class of monoids. We also give examples of monoids for which this is not the case, and in fact some nontrivial solution functions take arbitrary values at some points.

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