Publ. Math. Debrecen
In-print:: Ref. no.: 9538 (2024), 1–1

Exceptional sets related to the product of consecutive digits in Lüroth expansions

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Abstract. Every real number $x \in (0,1]$ admits a Lüroth expansion $[d_1(x), d_2(x), \ldots]_L$ with $d_n(x) \in \mathbb{N}_{\geq 2}$ being its digits. Let $\left\{\frac{p_n(x)}{q_n(x)}, n \geq 1\right\}$ be the sequence of convergents of the Lüroth expansion of x. We study the growth rate of the product of consecutive digits relative to the denominator of the convergent for the Lüroth expansion of an irrational number. More precisely, given a natural number m, we prove that the set

$$E_m(\beta) = \left\{x \in (0,1] \colon \limsup_{n\to\infty} \frac{\log\big(d_n(x)d_{n+1}(x)\cdots d_{n+m}(x)\big)}{\log q_n(x)} = \beta\right\}$$

and the set

$$\widetilde{E}_m(\beta) = \left\{x \in (0,1] \colon \limsup_{n\to\infty} \frac{\log\big(d_n(x)d_{n+1}(x)\cdots d_{n+m}(x)\big)}{\log q_n(x)} \geq \beta\right\}$$

share the same Hausdorff dimension for $\beta \geq 0$. It significantly generalises the existing results on the Hausdorff dimension of $E_1(\beta)$ and $\widetilde{E}_1(\beta)$.

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Mathematics Subject Classification: 11K55, 28A80, 11J83.
Key words and phrases: Lüroth expansion, Diophantine approximaion, Hausdorff dimension.