Publ. Math. Debrecen
In-print:: Ref. no.: 9666 (2025), 1–1

Non-transitive subgroups of co-rank one in the orthogonal group

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Abstract. It is known that any non-transitive closed subgroup G in the orthogonal group is remetrizable by a non-Euclidean Minkowski functional keeping the elements in G as linear isometries. Minkowski geometry is an alternative of Euclidean geometry for G. To measure the non-transitivity of the subgroup in the orthogonal group, we can use the Hausdorff distance between the Euclidean unit sphere and the orbit of a Euclidean unit element under G. The so-called flat subspace is spanned by the configuration of elements where the Hausdorff distance is attained at. Taking the maximum of the dimension of flat subspaces, we have the rank of the group G. It is known that subgroups of maximal rank are finite or reducible. They are natural prototypes of non-transitive subgroups in the orthogonal group. In the paper, we are going to investigate non-transitive subgroups of rank $n-1$ (co-rank one). We prove that the unit component $G^0 \subset G$ must be an Abelian subgroup in the special orthogonal group and the elements in G can be described in terms of subspaces given by simultaneous quasi-diagonalization in G^0. Independently of the rank condition, applications of simultaneous quasi-diagonalization are also presented for one- and two-dimensional non-transitive closed subgroups.

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Mathematics Subject Classification: 51B20, 52A40.
Key words and phrases: orthogonal group, non-transitive subgroups, orbits, Hausdorff distance.