

A note on variants of Euler's φ -function

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Abstract. It is well-known that the sum of the first n consecutive integers always divides the k -th power sum of the first n consecutive integers when k is odd. Motivated by this result, in this note we study the divisibility properties of the power sum of positive integers that are coprime to n and not surpassing n . First, we prove a finiteness result for our divisibility sets using smooth numbers in short intervals. Then, we find the exact structure of a certain divisibility set that contains the orders of these power sums and our result is of computational flavour.

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