BED property for the tensor product of Banach algebras

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Abstract. Let $\mathcal{A}$ and $\mathcal{B}$ be commutative and semisimple Banach algebras. Suppose that $\| \cdot \|_\gamma$ is an algebra cross-norm on $\mathcal{A} \otimes \mathcal{B}$ such that $\| \cdot \|_\gamma \geq \| \cdot \|_e$, and $\mathcal{A} \widehat{\otimes}_\gamma \mathcal{B}$ is a semisimple Banach algebra. In this paper, we verify the BED property for $\mathcal{A} \widehat{\otimes}_\gamma \mathcal{B}$. In fact, we show that if $\mathcal{A} \widehat{\otimes}_\gamma \mathcal{B}$ is of BED, then both $\mathcal{A}$ and $\mathcal{B}$ are so, whenever either $\mathcal{A}$ or $\mathcal{B}$ is unital. We also show that if $\mathcal{B}$ (resp., $\mathcal{A}$) is unital and $\widehat{\mathcal{A}} \subseteq C^0_{\mathrm{BSE}}(\Delta(\mathcal{A}))$ (resp., $\widehat{\mathcal{B}} \subseteq C^0_{\mathrm{BSE}}(\Delta(\mathcal{B}))$), then $\widehat{\mathcal{A} \widehat{\otimes}_\gamma \mathcal{B}} \subseteq C^0_{\mathrm{BSE}}(\Delta(\mathcal{A} \widehat{\otimes}_\gamma \mathcal{B}))$. We also establish that if $\mathcal{B}$ (resp., $\mathcal{A}$) is finite dimensional, then $\mathcal{A} \widehat{\otimes}_\gamma \mathcal{B}$ is of BED if and only if $\mathcal{A}$ (resp., $\mathcal{B}$) is of BED.

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