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On the explicit Galois group of $\mathbb{Q}(a_1^{rac{1}{m}},a_2^{rac{1}{m}},\ldots,a_n^{rac{1}{m}},\zeta_m)$ over \mathbb{Q}

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Abstract. Let m > 1 be an integer, and ζ_m denote a primitive *m*-th root of unity. For any finite set $S = \{a_1, a_2, \ldots, a_n\}$ and arbitrary choice of *m*-th roots of unity $\zeta_m^{r_i}$, $0 \leq r_i < m$ for $i = 1, \ldots, n$, we study the density of primes **p** of $\mathbb{Q}(\zeta_m)$ such that the *m*-th power residue symbol $\left(\frac{a_i}{\mathbf{p}}\right)_m = \zeta_m^{r_i}$. We calculate the explicit structure of the Galois group $\operatorname{Gal} \mathbb{Q}(a_1^{\frac{1}{m}}, \ldots, a_n^{\frac{1}{m}}, \zeta_m)/\mathbb{Q}(\zeta_m)$ in terms of its action on $a_i^{\frac{1}{m}}$ for $1 \leq i \leq n$.

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