Publ. Math. Debrecen
In-print:: Ref. no.: 9801 (2025), 1–1

On function SX of additive complements

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Abstract. Two sets A, B of nonnegative integers are called *additive complements*, if all sufficiently large integers can be expressed as the sum of two elements from A and B. We call A, B *perfect additive complements* if every nonnegative integer can be uniquely expressed as the sum of two elements from A and B. Let $A(x)$ be the counting function of A. In this paper, we focus on the function SX, where $SX = \limsup_{x\to\infty} \frac{\max\{A(x),B(x)\}}{\sqrt{x}}$ was introduced by Erdős and Freud in 1984. As a main result, we determine the value of SX for perfect additive complements, and further fix the infimum. We also give the absolute lower bound of SX for additive complements.

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Mathematics Subject Classification: Primary: 11B13; Secondary: 11B34.
Key words and phrases: additive complements, perfect, infimum.