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Uniqueness conjecture on simultaneous Pell equations. II

By YASUTSUGU FUJITA (Narashino) and MAOHUA LE (Zhanjiang)

Abstract. Let A and B be distinct positive integers. It is known that any positive solution to the simultaneous Pell equations $x^2 - Ay^2 = 1$ and $z^2 - By^2 = 1$ gives rise to a positive solution to the simultaneous Pell equations $x^2 - (m^2 - 1)y^2 = 1$ and $z^2 - (n^2 - 1)y^2 = 1$ for some distinct integers m and n greater than one. In this paper, we prove that the latter equations have only the positive solution $(x, y, z) = (m, 1, n)$ if $\{1, b, c\}$ is a Diophantine triple with $b = m^2 - 1$, $c = n^2 - 1$ and $c \geq \max\{200b^4, 2b^5\}$. Moreover, we show that the same conclusion holds if we replace the inequality assumed above by $b = \sigma p^e + 1$ for some prime p, a positive integer e and $\sigma \in \{1, 2, 4\}$.

YASUTSUGU FUJITA
DEPARTMENT OF MATHEMATICS
COLLEGE OF INDUSTRIAL TECHNOLOGY
NIHON UNIVERSITY, 2-11-1 SHIN-EI
NARASHINO, CHIBA
JAPAN

MAOHUA LE
INSTITUTE OF MATHEMATICS
LINGNAN NORMAL COLLEGE
ZHANJIANG, GUANGDONG, 524048
CHINA

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