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Rings whose nil-clean and clean elements are uniquely nil-clean

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Abstract. We consider and study those rings in which each nil-clean or clean element is uniquely nil-clean. We establish that those rings are abelian. More precisely, it is shown that the classes of abelian rings and the rings in which nil-clean elements are uniquely nil-clean do coincide. Moreover, we prove that the rings in which clean elements are uniquely nil-clean coincide with the subclass of abelian rings consisting of only unipotent units. In particular, we obtain in the semipotent case a complete characterization only in terms of the former ring and its divisions. Likewise, some extension properties and group rings for such kinds of rings are also considered.

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