Publ. Math. Debrecen
In-print:: Ref. no.: 10298 (2026), 1–1

Some properties of the quadrinomials $p(z) = 1 + \kappa(z + z^{N-1}) + z^N$ and $q(z) = 1 + \kappa(z - z^{N-1}) - z^N$

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Abstract. We show that all the zeros of the quadrinomial $p(z)=1+\kappa(z+z^{N-1})+z^N$ lie on the unit circle if and only if the inequalities

$$-1 \le \kappa \le \begin{cases} 1, & \text{if } N \text{ is } \textit{even}, \\ N/(N-2), & \text{if } N \text{ is } \textit{odd} \end{cases}$$

hold. For the quadrinomial $q(z) = 1 + \kappa(z - z^{N-1}) - z^N$, the corresponding inequalities are

$$-N/(N-2) \le \kappa \le \begin{cases} 1, & \text{if } N \text{ is } \textit{odd}, \\ N/(N-2), & \text{if } N \text{ is } \textit{even}. \end{cases}$$

In the cases of limiting values of the parameter κ, we provide factorization formulas for the corresponding quadrinomials. For example, when N is *odd* and $\kappa = N/(N-2)$, the following representation is valid: $p(z) = (1+z)^3 \prod_{j=1}^{(N-3)/2}[1 + z^2 - 2z\gamma_j]$, where $\gamma_j = 1 - 2\nu_j^2$ with $\{\nu_j\}_{j=1}^{(N-3)/2}$ being the collection of positive roots of the equation $U'_{N-2}(x) = 0$; here $U_j(x) = U_j(\cos t) = \frac{\sin(j+1)t}{\sin t} = 2^j x^j + \cdots$ are Chebyshev polynomials of the second kind, and $U'_j(x)$ are their derivatives. Similar factorization formulas are also provided for $q(z)$. As an application of the obtained results, we give the factorization formulas for the derivative of the Fejér polynomial, as well as construct certain univalent polynomials related to the polynomials $p(z)$ and $q(z)$.

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Mathematics Subject Classification: 26C10.
Key words and phrases: polynomials with zeros on the unit disk, Chebyshev polynomials of the second kind, derivative of the Chebyshev polynomial of the second kind, univalent polynomials.