

Cheng–Yau logarithmic gradient estimates for positive solutions to $\Delta_f u + p \frac{|\nabla u|^2}{u} + \mu(x)u^r = 0$ on smooth metric measure spaces

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Abstract. In this paper, through the combination of the Nash–Moser iteration and the Saloff-Coste-type Sobolev inequality, we establish Cheng–Yau logarithmic gradient estimates for solutions to the following nonlinear elliptic equation

$$\Delta_f u + p \frac{|\nabla u|^2}{u} + \mu(x)u^r = 0,$$

which is closely related to the gradient generalized τ -quasi-Einstein metrics. Especially, we obtain the Liouville property and Harnack inequality for this equation when $\mu(x)$ is constant.

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