

A note on the values of Barnes G -function at rational points

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Abstract. Let γ be the Euler’s constant, and $\psi(z)$ the digamma function. It is known that $\psi(z) + \gamma$ is transcendental whenever $z \in \mathbb{Q} \setminus \mathbb{Z}$. A classical function which comes close at the heels of the digamma function is the logarithmic derivative of the Barnes G -function, denoted by $\psi_G(z)$. Analogous to the above result on $\psi(z)$, we show in this paper that

$$\psi_G(z) - (1 - z)\gamma - \frac{1}{2} \log 2\pi$$

is transcendental whenever $z \in \mathbb{Q} \setminus \mathbb{Z}$. Further, under Schanuel’s conjecture, we show that the above result is true without the term $\frac{1}{2} \log 2\pi$, and at most one element from

$$\{\psi_G(z), z \in \mathbb{Q} \setminus \mathbb{Z}\}$$

is algebraic.

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Mathematics Subject Classification: 11J72, 11J81, 11J86.

Key words and phrases: digamma function, Barnes G -function, transcendence, Schanuel’s conjecture.