

The Diophantine equation $(2^k - 1)(b^k - 1) = y^q$

By CHANG LIU (Göttingen) and BO HE (Göttingen)

Abstract. In this paper, we consider the exponential Diophantine equation $(2^k - 1)(b^k - 1) = y^q$, with $k \geq 2$, an odd integer b , and an odd prime exponent q . We obtain effective upper bounds for q in terms of b . In particular, we show that $q \leq \log_2(b + 1)$ holds apart from a finite, explicitly determined set of exceptional pairs (b, q) with $3 \leq b < 10^6$. As an application, we prove that the related equation $(2^k - 1)(b^k - 1) = x^n$ has no positive integer solution (k, x, n) for several specific values of b , including $b \in \{5, 7, 11, 13, 21, 23, 27, 29\}$, except for $(2^2 - 1)(7^2 - 1) = 12^2$.

CHANG LIU
MATHEMATISCHES INSTITUT DER
UNIVERSITÄT GÖTTINGEN
BUNSENSTR. 3-5
DE-37073, GÖTTINGEN
GERMANY

BO HE
MATHEMATISCHES INSTITUT DER
UNIVERSITÄT GÖTTINGEN
BUNSENSTR. 3-5
DE-37073, GÖTTINGEN
GERMANY

AND
APPLIED MATHEMATICS INSTITUTE
ABA TEACHERS UNIVERSITY
WENCHUAN, SICHUAN, 623002
P. R. CHINA

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