Title: Discontinuous non-linear mappings on locally convex direct limits

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We show that the self-map $f : C_c^\infty(\mathbb{R}, \mathbb{R}) \to C_c^\infty(\mathbb{R}, \mathbb{R})$, $f(\gamma) := \gamma \circ \gamma - \gamma(0)$ of the space of real-valued test functions on the line is discontinuous, although its restriction to the space $C_K^\infty(\mathbb{R}, \mathbb{R})$ of functions supported in K is smooth (and hence continuous), for each compact subset $K \subseteq \mathbb{R}$. More generally, we construct mappings with analogous pathological properties on spaces of compactly supported smooth sections in vector bundles over non-compact bases. The results can be used in infinite-dimensional Lie theory to analyze the precise direct limit properties of test function groups and groups of compactly supported diffeomorphisms.

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