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Title: On the solvability of some special equations over finite fields

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Let F be a polynomial over $\mathbb{F}_p$ with n variables and of degree d. Suppose that it is impossible to transform F by invertible homogeneous linear change of variables to a polynomial, which has less than n variables. Also suppose that the degree of F in each variable is less than p. Rédei conjectured that if $d \le n$ then $F = 0$ has at least one solution in $\mathbb{F}_p$. This was disproved in [5] by a collection of counterexamples, but the cases $\deg F = 3$ and $\deg F = 5$ remained open. We give a counterexample with $\deg F = 5$ over $\mathbb{F}_{11}$. On the positive side, we prove the statement for symmetric polynomials of degree 3. Along a related line, consider polynomials of the form $F(x_1, \dots, x_n) = a_1 x_1^k + \dots + a_n x_n^k + g(x_1, \dots, x_n)$, where $a_1 a_2 \dots a_n \ne 0$, $g \in \mathbb{F}_p[x_1, \dots, x_n]$ and $\deg g < k$. We will show, that if $n \ge \left\lceil \frac{p-1}{\lfloor \frac{p-1}{k} \rfloor} \right\rceil$, then the equation $F(x_1, \dots, x_n) = 0$ is solvable in ${\mathbb{F}_p}^n$. This is a generalization of a result of Carlitz ([2]).

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