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Skew derivations with annihilating Engel conditions

By CHEN-LIAN CHUANG (Taipei), MING-CHU CHOU (Taipei)
and CHENG-KAI LIU (Taipei)

Abstract. Let R be a noncommutative prime ring and $a \in R$. Suppose that δ is a σ-derivation of R such that $a\big[\delta(x), x\big]_k = 0$ for all $x \in R$, where k is a fixed positive integer. Then $a = 0$ or $\delta = 0$ except when $R = M_2(GF(2))$.

1. Introduction and results

Throughout this paper, R is always a prime ring with center $Z(R)$. For $x, y \in R$, set $[x, y]_1 = [x, y] = xy - yx$ and $[x, y]_k = \big[[x, y]_{k-1}, y\big]$ for $k > 1$.

Let σ be an automorphism of R. A σ-derivation $\delta : R \to R$ is an additive map satisfying $\delta(rs) = \sigma(r)\delta(s) + \delta(r)s$ for all $r, s \in R$. For brevity, σ-derivations are generally called skew derivations. When σ is an identity map, σ-derivations are simply ordinary derivations. For $\sigma \neq 1$, the simplest example of σ-derivations is the $1 - \sigma$, where 1 denotes the identity map. Thus results of skew derivations are generalizations of both derivations and automorphisms.

For a subset S of R, a mapping $f : S \to R$ is called centralizing if $[f(x), x] \in Z(R)$ for all $x \in S$. In [19] Posner showed that R must be commutative if it possesses a nonzero centralizing derivation on R. In [17] Mayne proved the analogous result for nonidentity centralizing

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automorphisms. Many related generalizations have been obtained by a number of authors in the literature. See, for instance, [11]–[13], [18] and [21]. Recently, Filippis [7] proved the following: Let R be a prime ring of characteristic different from 2, d a nonzero derivation and L a noncentral Lie ideal of R. For $a \in R$, if $a[d(u), u] = 0$ for all $u \in L$, then $a = 0$. That is, the left annihilator of the set $\{[d(u), u] \mid u \in L\}$ is zero. Shiue [20] generalized this result by imposing the condition: $a[d(u), u]_k = 0$ for all $u \in L$, where k is a fixed positive integer. The main purpose of this article is to extend Shiue's result to skew derivations. Precisely, we will prove the following

Main Theorem. *Let R be a noncommutative prime ring and $a \in R$. Suppose that δ is a σ-derivation of R such that $a\big[\delta(x), x\big]_k = 0$ for all $x \in R$, where k is a fixed positive integer. Then $a = 0$ or $\delta = 0$ except when $R = M_2(GF(2))$.*

We give an example to show that the exceptional case indeed exists in the Main Theorem.

Example. Let $R = M_2(GF(2))$, $a = e_{11} + e_{12}$ and $\sigma(x) = gxg^{-1}$, where $g = e_{12} + e_{21}$. Let δ be a nonzero inner σ-derivation defined by $b = e_{21} + e_{22}$, that is $\delta(x) = \sigma(x)b - bx$. Then by a direct computation we have $a\big[[\delta(x), x], x\big] = 0$ for all $x \in R$.

2. Preliminaries

We denote by Q the two-sided Martindale quotient ring of R and by C the center of Q, which is called the extended centroid of R. Note that Q is also a prime ring and C is a field (see [1] for details).

A σ-derivation δ of Q is called inner if $\delta(x) = \sigma(x)b - bx$ for some $b \in Q$. Otherwise, it is said to be outer. An automorphism σ of Q is called inner if there exists an invertible $g \in Q$ such that $\sigma(x) = gxg^{-1}$ for all $x \in Q$. Otherwise, it is said to be outer. It is well-known that any automorphism of R can be uniquely extended to an automorphism of Q. Thus it is easy to verify that any σ-derivation of R can be uniquely extended to a σ-derivation of Q. An automorphism (a σ-derivation respectively) of R is called X-inner or X-outer according as its extension to Q is equal to an inner automorphism (or an inner σ-derivation respectively) of Q or not.

An automorphism σ of Q is called Frobenius if, in the case of char R=0, $\sigma(\alpha) = \alpha$ for all $\alpha \in C$ and if, in the case of char $R = p \geq 2$, $\sigma(\alpha) = \alpha^{p^n}$ for all $\alpha \in C$, where n is a fixed integer, positive, zero, or negative.

Theorem A. ([5]). *Let R be a prime ring with an X-outer σ-derivation δ. Then any generalized polynomial identity of R in the form $\phi(x_i, \delta(x_i)) = 0$ yields the generalized polynomial identity $\phi(x_i, y_i) = 0$ of R, where x_i, y_i are distinct indeterminates.*

Theorem B. ([3]). *Let R be a prime ring with an X-outer automorphism σ. Suppose that R satisfies a generalized polynomial identity $\phi(x_i, \sigma(x_i)) = 0$, where $\phi(x_i, y_i)$ is a nontrivial generalized polynomial in distinct indeterminates x_i, y_i. Then R is a GPI-ring.*

Theorem C. ([4]). *Let R be a prime ring with an automorphism σ. Suppose that σ is not an Frobenius automorphism of R. Then any generalized polynomial identity of R in the form $\phi(x_i, \sigma(x_i)) = 0$ yields the generalized polynomial identity $\phi(x_i, y_i) = 0$ of R, where x_i, y_i are distinct indeterminates.*

Theorem D. ([9, p. 140] or [1, Theorem 4.7.4]). *Let R be a prime GPI-ring with an automorphism σ and extended centroid C. Suppose that $\sigma(\alpha) = \alpha$ for all $\alpha \in C$. Then σ is an X-inner automorphism.*

Let V_D be a right vector space over a division ring D. An additive map $T \in \mathrm{End}(V)$ is called semi-linear transformation if for some automorphism τ of D, $T(v\alpha) = (Tv)\tau(\alpha)$ for all $v \in V$ and $\alpha \in D$ (see [8, p. 44]).

Theorem E. ([8, p. 79]). *Let R be a primitive ring with nonzero socle and ${}_RV$ a faithful irreducible left R-module. Let $D = \mathrm{End}({}_RV)$. Suppose that σ is an automorphism of R. Then there exists a semi-linear automorphism $T \in \mathrm{End}(V)$ such that $\sigma(r) = TrT^{-1}$ for all $r \in R$.*

Lemma F. ([15, Lemma 1.2]). *Let R be a prime ring and $a_i, b_i, c_j, d_j \in RC$. Suppose that $\sum_{i=1}^{m} a_i x b_i + \sum_{j=1}^{n} c_j x d_j = 0$ for all $x \in R$. If $a_1, \ldots, a_m$ are C-independent, then each b_i is a C-linear combination of $d_1, \ldots, d_n$.*

3. Proof of the Main Theorem

We first give a well-known lemma which has appeared in various papers.

Lemma 1. *Let V_D be a right vector space over a division ring D with $\dim V_D \geq 2$ and $T \in \mathrm{End}(V)$ such that v and Tv are D-dependent for every $v \in V$. Then there exists $\lambda \in D$ such that $Tv = v\lambda$ for all $v \in V$.*

PROOF. For each $v \in V$, we write $Tv = v\lambda_v$, where $\lambda_v \in D$. Pick a nonzero $v \in V$. For $w \in V$, if w and v are D-independent, then $(w+v)\lambda_{w+v} = T(w+v) = T(w) + T(v) = w\lambda_w + v\lambda_v$. So $w(\lambda_{w+v} - \lambda_w) = v(\lambda_v - \lambda_{w+v})$, and $\lambda_{w+v} = \lambda_w = \lambda_v$. If w and v are D-dependent, there exists $u \in V$ such that u and v are D-independent. So u and w are also D-independent. Then $\lambda_u = \lambda_v = \lambda_w$. We have done. □

The following lemma plays a key role in our proof.

Lemma 2. *Let R be a dense subring of the ring of linear transformations of a vector space V_D over a division ring D, containing nonzero linear transformations of finite rank, where $\dim V_D \geq 2$. Let σ be an automorphism of R. Suppose that $a, b \in R$ and $\delta(x) = \sigma(x)b - bx$ satisfy $a[\delta(x), x]_k = 0$ for all $x \in R$, where k is a fixed positive integer. Then $a = 0$ or $\delta = 0$ unless $\dim V_D = 2$ and $D = GF(2)$, the Galois field of two elements.*

PROOF. We assume $a \neq 0$ and $\delta \neq 0$ and proceed to show that $D = GF(2)$. Since R is a primitive ring with nonzero socle [8, p. 75], by Theorem E, there exists a semi-linear automorphism $T \in \mathrm{End}(V)$ such that $\sigma(x) = TxT^{-1}$ for all $x \in R$. Hence $a[TxT^{-1}b - bx, x]_k = 0$ for all $x \in R$.

We *claim* that there exists $v_0 \in V$ such that v_0 and $T^{-1}bv_0$ are D-independent: If not, then v and $T^{-1}bv$ are D-dependent for all $v \in V$. That is, for each $v \in V$ there exists $\lambda_v \in D$ such that $T^{-1}bv = v\lambda_v$. By Lemma 1, there exists $\lambda \in D$ such that $T^{-1}bv = v\lambda$ for all $v \in V$. Then

$$\begin{aligned}\delta(x)v &= (TxT^{-1}b - bx)v = T(xv\lambda) - bxv = T((xv)\lambda) - bxv \\ &= T(T^{-1}bxv) - bxv = 0,\end{aligned}$$

for all $x \in R$ and $v \in V$. Since V is faithful, we have $\delta = 0$, a contradiction. So v_0 and $T^{-1}bv_0$ are D-independent for some $v_0 \in V$.

First, assume $\dim V_D \geq 3$. Choose $w \in V$ such that w is D-independent of v_0 and $T^{-1}bv_0$. By the density of R, there exists $x \in R$ such that

$$xv_0 = 0, \quad xT^{-1}bv_0 = T^{-1}w, \quad xw = w.$$

This implies that

$$\begin{aligned} 0 = a[TxT^{-1}b - bx, x]_k v_0 &= a\sum_{i=0}^{k}(-1)^i\binom{k}{i}x^i(TxT^{-1}b - bx)x^{k-i}v_0 \\ &= (-1)^k ax^k TxT^{-1}bv_0 = (-1)^k ax^k w = (-1)^k aw \end{aligned}$$

and so $aw = 0$. Since $w+v_0$ is also D-independent of v_0 and $T^{-1}bv_0$, using $w+v_0$ instead of w, we also have $a(w+v_0)$=0. Similarly, $a(w+T^{-1}bv_0)$=0. So $av_0 = 0$ and $aT^{-1}bv_0 = 0$. Then $aV = 0$, a contradiction.

Second, assume $\dim V_D = 2$. Then v_0 and $T^{-1}bv_0$ form a basis of V_D. We *claim* that there exists $w \in V$ such that $w \notin v_0D$ and $Tw \notin v_0D$. Suppose on the contrary, for each $w \in V$, either $w \in v_0D$ or $w \in (T^{-1}v_0)D$. Then $V = v_0D \cup (T^{-1}v_0)D$. As a vector space cannot be the union of two proper subspace, we must have $\dim V_D = 1$, a contradiction. For such w with $w \notin v_0D$ and $w \notin (T^{-1}v_0)D$, write $w = v_0\alpha + (T^{-1}bv_0)\beta$ and $Tw = v_0\gamma + (T^{-1}bv_0)\ell$, where $\alpha, \beta, \gamma, \ell \in D$ and $\beta, \ell \neq 0$. By the density of R, there exists $x \in R$ such that $xv_0 = 0, xT^{-1}bv_0 = w$. This implies that $xw = x(v_0\alpha + (T^{-1}bv_0)\beta) = x(T^{-1}bv_0)\beta = w\beta$ and

$$xTw = x(v_0\gamma + (T^{-1}bv_0)\ell) = w\ell.$$

Then

$$\begin{aligned} 0 = a[TxT^{-1}b - bx, x]_k v_0 &= (-1)^k ax^k TxT^{-1}bv_0 \\ &= (-1)^k ax^k Tw = (-1)^k ax^{k-1}w\ell = (-1)^k aw\beta^{k-1}\ell \end{aligned}$$

and so $aw = 0$. If there exists a nonzero $\lambda \in D$ such that $T(w+v_0\lambda) \notin v_0D$, using $w+v_0\lambda$ instead of w, we have $a(w+v_0\lambda) = av_0\lambda = 0$ and so $av_0 = 0$. Since w and v_0 are D-independent and $\dim V_D = 2$, we have $aV = 0$, a contradiction. Thus $T(w + v_0\lambda) \in v_0D$ for all nonzero $\lambda \in D$. Suppose that $|D| > 2$. Choose two nonzero λ_1 and λ_2 in D with $\lambda_1 \neq \lambda_2$ such that $T(w + v_0\lambda_1) \in v_0D$ and $T(w + v_0\lambda_2) \in v_0D$. Then $T(v_0(\lambda_1 - \lambda_2)) = T(w + v_0\lambda_1) - T(w + v_0\lambda_2) \in v_0D$ and using semi-linearity of T, we have $T(v_0) \in v_0D$ and then $T(w) \in v_0D$, a contradiction. The proof is now complete. □

Proof of Main Theorem. We may assume $a \neq 0$ and $\delta \neq 0$ and proceed to show that $R = M_2(GF(2))$. Suppose that δ is X-outer. By Theorem A, we have $a[y, x]_k = 0$ for all $x, y \in R$. Pick $b \in R\backslash C$ and replace y by $xb - bx$. Then $a[d(x), x]_k = 0$ for all $x \in R$, where $d(x) = xb - bx$ is a nonzero X-inner derivation. Hence we may assume that δ is X-inner and write $\delta(x) = \sigma(x)b - bx$ for some $b \in Q$.

Case 1. Suppose that σ is X-inner. Thus there exists an invertible element $g \in Q$ such that $\sigma(x) = gxg^{-1}$. Note that $g^{-1}b \notin C$. If $g^{-1}b \in C$, then $\delta(x) = gxg^{-1}b - bx = g(xg^{-1}b - g^{-1}bx) = g[x, g^{-1}b] = 0$, a contradiction. With this, we can see easily that

$$f(x) = a\big[\sigma(x)b - bx, x\big]_k = a\sum_{i=0}^{k-1}(-1)^i\binom{k}{i}x^i(gxg^{-1}b - bx)x^{k-i}$$
$$+ (-1)^k ax^k(-b)x + (-1)^k ax^k gxg^{-1}b$$

is a nontrivial GPI of R, since $g^{-1}b \notin C$ and $a \neq 0$. By [2], $f(x)$ is also a GPI of Q. Denote by F the algebraic closure of C or C according as C is infinite or finite respectively. By a standard argument [14, Proposition], $f(x)$ is also a GPI of $Q \otimes_C F$. Since $Q \otimes_C F$ is a centrally closed prime F-algebra [6, Theorem 3.5], by replacing R, C with $Q \otimes_C F$ and F respectively, we may assume that R is centrally closed and the field C is either algebraically closed or finite. By Martindale's Theorem [16, Theorem 3], R is a primitive ring having nonzero socle with the field C as its associated division ring. By [8, p. 75] R is isomorphic to a dense subring of the ring of linear transformations of a vector space V over C, containing nonzero linear transformations of finite rank. Since R is not commutative, we may assume $\dim_C V \geq 2$. By Lemma 2, we are done in this case.

Case 2. Suppose that σ is X-outer. We first claim that if $a \neq 0$ and $b \neq 0$, then R is a GPI-ring: Observe that $a\big[yb - bx, x\big]_k = a\big[yb, x\big]_k - a\big[bx, x\big]_k$ is a nontrivial generalized polynomial. Thus $a[\sigma(x)b - bx, x]_k = 0$

is a nontrivial GPI of R. So R is a GPI-ring follows from Theorem B. By [4], $a[\sigma(x)b-bx,x]_k=0$ is also a GPI of Q. By MARTINDALE's Theorem [16], Q is a primitive ring having nonzero socle and its associated division ring D is finite-dimensional over C. Hence Q is isomorphic to a dense subring of the ring of linear transformations of a vector space V over D, containing nonzero linear transformations of finite rank. If $\dim V_D\geq 2$, by Lemma 2, we are done. So we may assume $\dim V_D=1$, that is, $Q\cong D$. If C is finite, then $\dim D_C<\infty$ implies that D is also finite. Thus D is a field by Wedderburn's Theorem [8, p. 183] on finite division rings. In particular, Q is commutative, a contradiction. Hence from now on we assume C is infinite and Q is a division ring. By assumption $a\neq 0$, we have $[\sigma(x)b-bx,x]_k=0$ for all $x\in R$.

Subcase 1. Suppose that σ is not Frobenius. Then by Theorem C, $[yb-bx,x]_k=0$ for all $x,y\in R$. Taking $y=x$, we have $[b,x]_{k+1}=0$ for all $x\in R$. By [12], it follows that $b\in C$. Then $0=[yb-bx,x]_k=b[y-x,x]_k=b[y,x]_k$. Thus $0=[y,x]_k$ for all $x,y\in R$. So $y\in C$ for all $y\in R$ by [12] again. Hence R is commutative, a contradiction.

Subcase 2. Suppose that σ is Frobenius. For simplicity, we denote x^σ by $\sigma(x)$. We may assume that $\operatorname{char} R=p>0$. Otherwise, if $\operatorname{char} R=0$, then the Frobenius automorphism σ fixes C and hence must be X-inner by Theorem D, a contradiction. So for all $\alpha\in C$, $\alpha^\sigma=\alpha^{p^n}$ for some nonzero fixed integer n. Also we may assume $n\neq 0$ by Theorem D. By [4], $[\sigma(x)b-bx,x]_k=0$ for all $x\in Q$. Replacing x by $x+\alpha$, where $0\neq\alpha\in C$, we have

$$\begin{aligned}0&=[(x+\alpha)^\sigma b-b(x+\alpha),x+\alpha]_k=\big[\big(x^\sigma+\alpha^{p^n}\big)b-b(x+\alpha),x\big]_k\\&=[b,x]_k\alpha^{p^n}-[b,x]_k\alpha+[x^\sigma b-bx,x]_k=[b,x]_k\alpha^{p^n}-[b,x]_k\alpha.\end{aligned}$$

If $[b,x]_k\neq 0$ for some $x\in Q$, we see that $\alpha^{p^n}=\alpha$ for all $\alpha\in C$. So C is finite, a contradiction. Hence $[b,x]_k=0$ for all $x\in Q$. By [12], we have $b\in C$ and then $0=[x^\sigma b-bx,x]_k=b[x^\sigma-x,x]_k=b[x^\sigma,x]_k$. Thus $0=[x^\sigma,x]_k$. Since there exists integer m such that $p^m>k$, we have that $[x^\sigma,x]_{p^m}=0$. It follows that $\big[x^\sigma,x^{p^m}\big]=0$ for all $x\in Q$, since $\operatorname{char} R=p>0$.

Suppose first $n\geq 1$. For $\alpha\in C$ and $y\in Q$, replacing x by $x+\alpha y$, we have $0=\big[(x+\alpha y)^\sigma,(x+\alpha y)^{p^m}\big]=\big[x^\sigma+\alpha^{p^n}y^\sigma,\sum_{i=0}^{p^m}\phi_i(x,y)\alpha^i\big]$, where

$\phi_i(x,y)$ denotes the sum of all monic monomials with x-degree $p^m - i$ and y-degree i for $0 \le i \le p^m$. In particular, $\phi_1(x,y) = x^{p^m-1}y + x^{p^m-2}yx + \cdots + yx^{p^m-1} = \sum_{i=0}^{p^m-1} x^{(p^m-1-i)}yx^i$. As C is infinite, it follows from the Vander Monde determinant argument that $[x^\sigma, \phi_1(x,y)] = 0$. Hence

$$x^\sigma \sum_{i=0}^{p^m-1} x^{(p^m-1-i)}yx^i - \sum_{i=0}^{p^m-1} x^{(p^m-1-i)}yx^i x^\sigma = 0 \tag{1}$$

for all $x, y \in Q$. Given $x \in Q$, if $\phi_1(x,y)$ is an identity of Q, that is, $\phi_1(x,y) = 0$ for all $y \in Q$, then $0 = [x, \phi_1(x,y)] = \left[x, \sum_{i=0}^{p^m-1} x^{(p^m-1-i)}yx^i\right] = [x^{p^m}, y]$ for all $y \in Q$. Thus $x^{p^m} \in C$. If $x^{p^m} \in C$ for all $x \in R$, then Q is a field by [8, p. 185, Theorem 3] and R is commutative, a contradiction. We may thus choose $x \in Q$ such that $x^{p^m} \notin C$ and for this x, $\phi_1(x,y)$ is not an identity in y. Let $1 \le l \le p^m - 1$ be the maximal integer such that $1, x, \ldots, x^l$ are C-independent. Write $\phi_1(x,y) = \sum_{i=0}^{l} x^i y g_i(x)$, where $g_i(x)$ are polynomials in $1, x, \ldots, x^l$ over C. Note that $g_s(x) \neq 0$ for some s, since $\phi_1(x,y)$ is not an identity in y. Rewrite (1) in a form that

$$x^\sigma \sum_{i=0}^{p^m-1} x^{(p^m-1-i)}yx^i - \sum_{i=0}^{l} x^i y g_i(x) x^\sigma = 0,$$

for all $y \in Q$. By Lemma F, $g_i(x)x^\sigma$ are C-linear combinations of $1, x, \ldots, x^{p^m-1}$ for $i = 0, \ldots, l$. Since $g_s(x) \neq 0$ for some s and Q is a division ring, we also have x^σ is the C-linear combination of $\{g_s(x)^{-1}x^i\}$. Hence $[x^\sigma, x] = 0$. For any $y \in Q$, there exist infinite many $\beta \in C$ such that $(x+\beta y)^{p^m} \notin C$. Thus $0 = [(x+\beta y)^\sigma, x+\beta y] = [x^\sigma + \beta^{p^n}y^\sigma, x+\beta y]$. By the Vander Monde determinant argument again, $[x^\sigma, y] = 0$ for all $y \in Q$. Then $x^\sigma \in C$. Hence $x \in C$ and so $x^{p^m} \in C$, a contradiction.

Suppose next that $n \le -1$. Recall that $\left[x^\sigma, x^{p^m}\right] = 0$ for all $x \in Q$. Similarly, replacing x by $x + \alpha y$, we have $0 = \left[(x+\alpha y)^\sigma, (x+\alpha y)^{p^m}\right] = \left[x^\sigma + \alpha^{p^n}y^\sigma, \sum_{i=0}^{p^m} \phi_i(x,y)\alpha^i\right]$, where $\phi_i(x,y)$ denotes the sum of all monic monomials with x-degree $p^m - i$ and y-degree i for $0 \le i \le p^m$. Then $\left[\alpha^{p^{-n}}x^\sigma + y^\sigma, \sum_{i=0}^{p^m} \phi_i(x,y)\alpha^i\right] = 0$. As C is infinite, it follows from the Vander Monde determinant argument that $\left[y^\sigma, x^{p^m}\right] = 0$ for all $x, y \in R$. Thus Q is commutative by [12], a contradiction. The proof is now complete. □

References

[1] K. I. Beidar, W. S. Martindale 3rd and A. V. Mikhalev, Rings with Generalized Identities, *Marcel Dekker, Inc.*, *New York – Basel – Hong Kong*, 1996.

[2] C. L. Chuang, GPIs having coefficients in Utumi quotient rings, *Proc. Amer. Math. Soc.* **103** (1988), 723–728.

[3] C. L. Chuang, Differential identities with automorphisms and antiautomorphisms I, *J. Algebra* **149** (1992), 371–404.

[4] C. L. Chuang, Differential identities with automorphisms and antiautomorphisms II, *J. Algebra* **160** (1993), 292–335.

[5] C. L. Chuang and T. K. Lee, Identities with a single skew derivation, (*to appear in* J. Algebra).

[6] T. S. Erickson, W. S. Martindale 3rd and J. M. Osborn, Prime nonassociative algebras, *Pacific J. Math.* **60** (1975), 49–63.

[7] V. De Filippis, On the annihilator of commutators with derivation in prime rings, *Rend. Del Circ. Math. Di Palermo, Serie II* **49** (2000), 343–352.

[8] N. Jacobson, Structure of rings, Vol. 37, Amer. Math. Soc., *Colloq. Pub.*, *Rhode Island*, 1964.

[9] V. K. Kharchenko, Generalized identities with automorphisms, *Algebra i Logika* **14**(2) (1975), 215–237; Engl. Transl.: *Algebra and Logic* **14**(2) (1975), 132–148.

[10] V. K. Kharchenko and A. Z. Popov, Skew derivation of prime rings, *Comm. Algebra* **20** (1992), 3321–3345.

[11] C. Lanski, An Engel condition with derivation, *Proc. Amer. Math. Soc.* **118** (1993), 75–80.

[12] C. Lanski, An Engel condition with derivation for left ideals, *Proc. Amer. Math. Soc.* **125** (1997), 339–345.

[13] P. H. Lee and T. K. Lee, Lie ideals of prime rings with derivations, *Bull. Inst. Math. Acad. Sinica* **11** (1983), 75–80.

[14] P. H. Lee and T. L. Wong, Derivations cocentralizing Lie ideals, *Bull. Inst. Math. Acad. Sinica* **23** (1995), 1–5.

[15] P. H. Lee, T. L. Wong, J. S. Lin and R. J. Wang, Commuting traces of multiadditive mappings, *J. Algebra* **193** (1997), 709–723.

[16] W. S. Martindale III, Prime rings satisfying a generalized polynomial identity, *J. Algebra* **12** (1969), 576–584.

[17] J. H. Mayne, Centralizing automorphisms of prime rings, *Canad. Math. Bull.* **19** (1976), 113–115.

[18] J. H. Mayne, Centralizing mappings of prime rings, *Canad. Math. Bull.* **27** (1984), 122–126.

[19] E. C. Posner, Derivations in prime rings, *Proc. Amer. Math. Soc.* **8** (1957), 1093–1100.

[20] W. K. Shiue, Annihilators of derivations with Engel conditions, *Rend. Del Circ. Math. Di Palermo, Serie II,* **52** (2003), 505–509.

[21] J. Vukman, Commuting and centralizing mappings in prime rings, *Proc. Amer. Math. Soc.* **109** (1990), 47–52.

CHEN-LIAN CHUANG
DEPARTMENT OF MATHEMATICS
NATIONAL TAIWAN UNIVERSITY
TAIPEI 106
TAIWAN

E-mail: chuang@math.ntu.edu.tw

MING-CHU CHOU
DEPARTMENT OF MATHEMATICS
NATIONAL TAIWAN UNIVERSITY
TAIPEI 106
TAIWAN

E-mail: bella@vot2.url.com.tw

CHENG-KAI LIU
DEPARTMENT OF MATHEMATICS
NATIONAL TAIWAN UNIVERSITY
TAIPEI 106
TAIWAN

E-mail: ckliu@math.ntu.edu.tw

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