Title: A characterization of the identity function with an equation of Hosszú type

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The functional equation

$$f : \mathbb{R} \to \mathbb{R}, \ f(x + y - xy) + f(xy) = f(x) + f(y) \quad \text{for all } x, y \in \mathbb{R}$$

was first presented by M. Hosszú (1967) and now it is referred to as the Hosszú equation. The aim of this note is to consider an equation of Hosszú type on the domain $\mathbb{N}$. We prove that if a completely multiplicative function f satisfies the equation

$$f(p + q + pq) = f(p) + f(q) + f(pq)$$

for all primes p, q and $f(p_0) \neq 0$ for some prime number p_0, then $f(n) = n$ for all positive integers n.

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