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Finite groups with many values in a column or a row of the character table

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This paper is dedicated to the memory of Dr. Edith Szabó

Abstract. Many results show how restrictions on the values of the irreducible characters on the identity element (that is, the degrees of the irreducible characters) of a finite group G, influence the structure of G. In the current article we study groups with restrictions on the values of a nonidentity rational element of the group. More specifically, we show that S_3 is the only nonabelian finite group that contains a rational element g such that $\chi_1(g) \neq \chi_2(g)$ for all distinct $\chi_1, \chi_2 \in \mathrm{Irr}(G)$. We comment that the dual statement is also true: S_3 is the only finite nonabelian group that has a rational irreducible character that takes different values on different conjugacy classes.

1. Introduction

There are no finite groups in which all the irreducible characters have distinct degrees (see, e.g., [1]). Our first result is a variation of this situation, in which we replace the degrees by "another" rational column of the character table. We show that:

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Theorem 1. *Let G be a finite nonidentity group with a rational element g such that $\chi_1(g) \neq \chi_2(g)$ for every distinct $\chi_1, \chi_2 \in \mathrm{Irr}(G)$. Then $G \simeq S_2$, or S_3.*

A dual statement is:

Theorem 2. *Let G be a finite nonidentity group with a rational $\chi \in \mathrm{Irr}(G)$ such that $\chi(g_1) \neq \chi(g_2)$ for any non-conjugate elements $g_1, g_2 \in G$. Then $G \simeq S_2$ or S_3.*

Compare this to the conjecture (proved for solvable groups in [14] and independently in [12]) that S_3 is the only finite group for which the conjugacy character $x \to |C_G(x)|$ takes on different values on different conjugacy classes.

Our notation is standard (see [9]). We use the notation $\mathrm{class}_G(x)$ for the conjugacy class of the element x in the group G. The set $\mathrm{Lin}(G)$ is the set of all linear characters of G.

2. Proof of Theorem 1

Proof. Let G be a counter-example of minimal order. If $1 \neq M \lhd G$ where M is a subgroup with $g \notin M$, then $\theta(gM) \neq \phi(gM)$ for every distinct $\theta, \phi \in \mathrm{Irr}(G/M)$. By induction we get that $G/M \cong S_2$ or S_3.

First we show that G is a rational group. Indeed, if $\chi \in \mathrm{Irr}(G)$ then $\chi^\sigma \in \mathrm{Irr}(G)$ for all $\sigma \in Gal(\mathbb{Q}(\sqrt[|G|]{1})/\mathbb{Q})$. Since g is rational we have that $\chi^\sigma(g) = \chi(g)$, and by our assumption we get that $\chi^\sigma = \chi$ for all $\chi \in \mathrm{Irr}(G)$. Thus G is rational.

Assume first that $G = G'$. Then G has a proper normal subgroup N such that G/N is a nonabelian rational simple group. By [6], $G/N \cong SP(6,2)$ or $O_8^+(2)'$. If $g \in N$, then $g \in \mathrm{Ker}(\eta)$ for all $\eta \in \mathrm{Irr}(G/N)$. Each of the groups $SP(6,2)$ and $O_8^+(2)'$ have two distinct irreducible characters χ_1 and χ_2 of the same degree (see [4]), so $N = \ker(\chi_i)$ and we get that $\chi_1(g) = \chi_1(1) = \chi_2(1) = \chi_2(g)$, contradicting our assumption. Thus $g \notin N$. Since $G/N \ncong S_2$, S_3 we get that $N = 1$. Again, [4] shows neither $SP(6,2)$ nor $O_8^+(2)'$ has an element on which different irreducible characters take on distinct values. So we may assume that $G \neq G'$. Note that $g \notin G'$ because otherwise at least two linear characters will include g in their kernel and so both will have value 1 on g.

Let $\lambda \in \mathrm{Lin}(G) - \{1_G\}$. Since $1_G(g) = 1$, we get that $\lambda(g)$ is a rational root of unity different from 1. Thus $\lambda(g) = -1$ for all $\lambda \in \mathrm{Lin}(G) - \{1_G\}$. Our assumption now implies that $\mathrm{Lin}(G) = \{1_G, \lambda\}$, so that $|\mathrm{Lin}(G)| = |G : G'| = 2$. In particular, $G = G'\langle g\rangle$. Then $\lambda(y) = \lambda(yG') = \lambda(gG') = -1$ for all $y \in G - G'$.

Let $\chi \in \mathrm{Irr}(G) - \mathrm{Lin}(G)$. There are two possibilities, either $\lambda\chi = \chi$ or $\lambda\chi \neq \chi$. If $\lambda\chi = \chi$, then $\lambda(g)\chi(g) = -\chi(g) = \chi(g)$ and so $\chi(g) = 0$. Our assumption implies that there is at most one $\chi \in \mathrm{Irr}(G)$ with $\lambda\chi = \chi$ and this χ, if exists, vanishes on g (and in fact on $G - \ker\lambda = G - G'$).

The second possibility is that $\lambda\chi \neq \chi$. All but possibly one member of $\mathrm{Irr}(G)$ satisfy this. No χ with $\lambda\chi \neq \chi$ can vanish on g, because otherwise there will be distinct irreducible characters $\lambda\chi$, χ vanishing on g, contrary to our assumption.

In this case

$$\lambda\chi(y) = \begin{cases} \chi(y) & \text{if } y \in G' = \ker(\lambda) \\ -\chi(y) & \text{if } y \notin G'. \end{cases}$$

We show that this implies that $\chi_{G'} \in \mathrm{Irr}(G')$. For if not then $\chi_{G'} = \alpha_1 + \alpha_2$ with $\alpha_1, \alpha_2 \in \mathrm{Irr}(G')$ two distinct characters, and α_1 is not G-invariant. So the inertia group $I_G(\alpha_1) < G$ forcing $I_G(\alpha_1) = G'$. It follows ([9] p. 95 problem 1) that $(\alpha_1)^G$ is irreducible, with χ an irreducible constituent. Hence $(\alpha_1)^G = \chi$. Thus χ vanishes on $G - G'$ and in particular $\chi(g) = 0$, a contradiction.

We can now write $\mathrm{Irr}(G) = \{1_G, \lambda, \chi, \chi_1, \lambda\chi_1, \chi_2, \lambda\chi_2, \ldots, \chi_s, \lambda\chi_s\}$ where s is a non-negative integer. Note that χ may not exist, but if it does, it vanishes on $G - G'$. The other χ_i's and $\lambda\chi_i$'s restrict irreducibly to G' and never vanish on g. Also χ (if exists) either restricts irreducibly or to a sum of two irreducible characters of G'. Since each element of $\mathrm{Irr}(G')$ is a constituent of a restriction of some member of $\mathrm{Irr}(G)$ we get that all but possibly two members of $\mathrm{Irr}(G') - \{1_{G'}\}$ are G-invariant. By the Brauer permutation lemma ([9], Theorem 6.23, p. 93), all but possibly two nonidentity G'-conjugacy classes are in fact G-conjugacy classes.

Suppose that $s = 0$. If χ does not exist, then $G \cong S_2$, a contradiction. If χ does exist, G has exactly three conjugacy classes so that $G \cong S_3$ (see, e.g., [13]), a contradiction again. So $s \geq 1$. In particular $G' > 1$.

We now break the proof into two cases: $G' = G''$ and $G' \neq G''$.

Case 1. $G' = G''$.

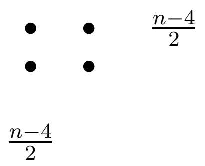
Let L be a proper subgroup of G' maximal subject to being normal in G. Then G'/L is a minimal normal subgroup of G/L. As G' has no abelian factor groups, we get that $G'/L = T_1 \times T_2 \times \ldots \times T_r$ where the T_i's are isomorphic nonabelian simple groups. In particular $G/L \ncong S_2, S_3$. Induction implies that $L = 1$, so that $G' = T_1 \times T_2 \times \ldots \times T_r$. Set $T = T_1$. As $T \lhd G'$, we have that $T^g \lhd G'$. Suppose that $T \neq T^g$, then $T \cap T^g \lhd T$ and as T is simple we get that $T \cap T^g = 1$. Since $G = G' \langle g \rangle$ with $g^2 \in G'$, we get that $T \times T^g \lhd G$. But G' is a minimal normal subgroup of G, so $G' = T \times T^g$. As T is a nonabelian simple group, $|T|$ has at least three prime divisors, say p_1, p_2, p_3. Fix $p = p_i$ and let $x_p \in T$ be an element of order p. Then $x_p^g \in T^g$ and so $class_G(x_p) \subsetneq T$ (since $T \cap T^g = 1$). However, $G' = T \times T^g$ and $T^g \subset C_{G'}(x_p)$, so every G'-conjugate of x_p has the form x_p^t for some $t \in T$ and hence $class_{G'}(x_p) \subset T$. So G' has more than two nonidentity G'-conjugacy classes which are not G-conjugacy classes. This is a contradiction as we have at most two such classes in G'. We conclude that $T = T^g$ and so G' is a nonabelian simple group.

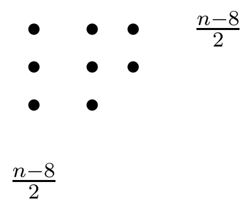
Suppose that $C = C_G(G') \neq 1$. Then $C \lhd G$ so that $C \cap G' \lhd G'$. As G' is a nonabelian simple group, either $C \cap G' = G'$ or $C \cap G' = 1$. In the former case $G' \subseteq C = C_G(G')$ which is impossible as G' is nonabelian. Thus $C \cap G' = 1$ and as $|G : G'| = 2$ we conclude that $G = C \times G'$ with $|C| = 2$. It follows that $G' \cong G/C$ is a rational nonabelian simple group, so by [6] $G' \cong SP(6,2)$ or $O_8^+(2)'$. In particular $G/C \ncong S_2$ or S_3, and induction implies that $g \in C = \ker(\sigma)$ for all $\sigma \in \mathrm{Irr}(G/C)$. Each of the groups $SP(6,2)$ and $O_8^+(2)'$ have two distinct irreducible characters χ_1 and χ_2 of the same degree (see [4]), so $g \in \ker(\chi_i)$ and we get that $\chi_1(g) = \chi_1(1) = \chi_2(1) = \chi_2(g)$, contradicting our assumption.

Thus $C_G(G') = 1$ and so $G \subset Aut(G')$. Recall that all but at most one element of $\mathrm{Irr}(G) - \{1_G\}$ restrict irreducibly to G'.

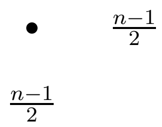
Since G is a rational group, [6] implies that G' is isomorphic to one of the groups: A_n, $n > 4$; $PSP(4,3)$; $SP(6,2)$, $SO^+(8,2)$, $PSL(3,4)$, $PSU(3,4)$. We now discuss each of these cases.

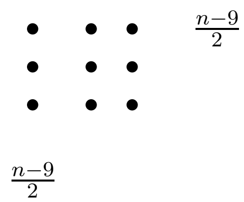
Suppose that $G' = A_n$. Suppose first that $n \geq 8$. Then $G = S_n$. Now by [10] p. 66, an irreducible character of S_n does not restrict irreducibly to A_n if and only if the partition of n corresponding with it is self-associate. For n even the following partitions are self associate:





For n odd the following partitions are self associate:





So we have at least two nonlinear irreducible characters of G which do not restrict irreducibly to G', a contradiction. For $n = 5, 6, 7$, S_n does not satisfy the assumption of the theorem (see tables of [10], p. 349).

If $G' \cong Sp(6,2)$, then $Out(G') = 1$, a contradiction. If $G' \cong PSP((4,3)$, $SO^+(8,2)$, $PSL(3,4)$ or $PSU(3,4)$ then by looking in the fusion column for each of the groups of the type $G' \cdot 2$ in [4], we see that each has at least two nonlinear irreducible characters which do not restrict irreducibly to G', a contradiction.

Case 2. $G' \neq G''$.

For each i, the characters χ_i and $\lambda\chi_i$ are rational, non-linear and restrict irreducibly to G', so their restrictions to G' are rational and nonlinear. However $G' \neq G''$ and so G' must have nonprincipal linear characters.

As each element of $\mathrm{Irr}(G')$ is a constituent of a restriction of some element of $\mathrm{Irr}(G)$, and since $\lambda_{G'} = 1_{G'}$ we get that χ does exist and $\chi_{G'}$ must have a nonprincipal linear constituent. As χ is nonlinear, $\chi_{G'}$ is reducible and so $\chi_{G'} = \delta_1 + \delta_2$ with $\delta_i \in \mathrm{Lin}(G')$ and hence $\mathrm{Lin}(G') = \{1_{G'}, \delta_1, \delta_2\}$. So $|G' : G'| = 3$ and $G/G' \cong S_3$.

Now take $u \in G' - G''$. Then $\delta_i(u) = \delta_i(uG'')$ is a cubic root of unity. It particular $\delta_i(u)$ is not rational. So δ_1, δ_2 are not rational, and all other elements of $\mathrm{Irr}(G')$ are rational. So every irreducible nonlinear character of G' is rational. Next take χ_i for some i. Then $\chi_i|_{G'}$ is an irreducible nonlinear rational character of G'. Therefore $\delta_1\chi_i|_{G'}$ is also an irreducible nonlinear character of G'. So $\delta_1\chi_i|_{G'}$ is rational. Thus $\delta_1\chi_i|_{G'}(u) = \delta_1(u)\chi_i(u)$ is rational. But $\chi_i(u)$ is rational and $\delta_1(u)$ is not rational. This implies that $\chi_i(u) = 0$. Clearly $\lambda\chi_i(u) = 0$ as well.

It follows that every nonlinear character of G' vanishes on u. Hence

$$|C_{G'}(u)| = |1_{G'}(u)|^2 + |\delta_1(u)|^2 + |\delta_2(u)|^2 = 3.$$

So u commutes in G' only with its powers. In particular $|u| = 3$ and u acts with no fixed points on G''. Thus the group $G' = \langle u \rangle G''$ is a Frobenius group with G'' the Frobenius kernel. A theorem of Thompson implies that G'' is nilpotent. In particular, $Z(G'') \neq 1$. By induction we get that $G/Z(G'') = S_2$ or S_3. But $|G/Z(G'')| \geq |G/G''| = 6$. We conclude that $G/Z(G'') = S_3$, so $|G/Z(G'')| = |G/G''| = 6$ which implies that $G'' = Z(G'')$, namely, G'' is abelian.

By a theorem of Ito ([9], Theorem 6.15, p. 84) every irreducible character of G' has degree dividing $|G'/G''| = 3$. So $\chi_i(1) = 3$ for all i.

So $\mathrm{Irr}(G)$ contains two linear characters, one character of degree 2, and all the rest have degree 3.

Let us get back to the element $g \in G - G'$ on which the irreducible characters assume distinct values. Fix an i. As $\chi_i(1) = 3$ the rational number $\chi_i(g)$ is a (nonzero) sum of three roots of unity. As $|\chi_i(g)| \leq 3$ we get that $\chi_i(g) = -3,\ 3, -2, 2, -1, 1$. Since $\lambda(g) = -1$ and $1_G(g) = 1$ we obtain that $\chi_i(g) = -3, 3, -2, 2$.

If $\chi_i(g) = a$ then $\lambda\chi_i(g) = -a$. If $s > 2$ let $\chi_1(g) = a_1$, $\chi_2(g) = a_2$, $\chi_3(g) = a_3$ where $a_1, a_2, a_3, -a_1, -a_2, -a_3$ are six different rationals on one hand, and each has to be either -3, $3, -2$ or 2 on the other hand. This is a contradiction. It follows that $s \leq 2$. If $s = 1$ then G has five conjugacy classes, and if $s = 2$ then G has seven conjugacy classes (two linear characters, s χ_i's, s $\lambda\chi_i$'s and one χ). We now use [13]. From the list of groups G with five or seven conjugacy classes only S_4 and S_5 are rational groups satisfying $|G : G'| = 2$. But $(S_5)'' = (S_5)'$ and S_4 does not satisfy the assumption of the theorem. This is the final contradiction. □

3. Proof of Theorem 2

Proof. Recall that $\chi \in \mathrm{Irr}(G)$ is rational, and $\chi(g_1) \neq \chi(g_2)$ for any non-conjugate elements $g_1, g_2 \in G$. Let $y_1, y_2 \in G^\#$ be such that $\langle y_1 \rangle = \langle y_2 \rangle$. By Lemma 5.22 of [9], $\chi(y_1) = \chi(y_2)$, so our assumption implies (among other things) that y_1, y_2 are conjugate. Thus G is a rational group. Our assumption implies that χ is faithful.

If $|C_G(y)| = 2$ for some $y \in G$ then G is a Frobenius group with an abelian Frobenius kernel K of odd order and a Frobenius complement of order 2 (see, e.g., Lemma 2.3 of [3]). Thus, every nonlinear character of G has degree equal to 2, and it vanishes outside K. It follows that χ is nowhere zero on K and $\chi(1) = 2$. Let $w \in K^\#$, then $|\chi(w)| \leq \chi(1) = 2$ and since $\chi(w)$ is rational, the assumption implies that $\chi(w) = -2, -1, 1$. Thus $K^\#$ is a union of no more than three G-conjugacy classes, each of size equal to two. Hence $|K| \leq 7$. However, if $|K| = 5$ or 7, then G is not rational, so $|K| = 3$ and $G \simeq S_3$.

Therefore we may assume that $|C_G(y)| > 2$ for all $y \in G$.

If χ is linear then G has at most two conjugacy classes, so $G \simeq S_2$.

Thus we may assume that χ is nonlinear, so G is non-abelian and χ must vanish on some element of G. Suppose that C is the unique conjugacy class of G on which χ vanishes. Further, if χ assumes the value 1 we denote by D the unique conjugacy class of G on which χ takes on the value 1. Similarly, if χ assumes the value -1 we denote by E the unique conjugacy class of G on which χ takes on the value -1. If D (respectively E) does not exist we set $D = \emptyset$ (respectively $E = \emptyset$). A variation of a theorem of

Thompson ([2] p. 147) now implies that $|C \cup D \cup E| \geq \frac{3}{4}|G|$, and equality forces $|G| = 8$. As the only nonlinear irreducible character of a non-abelian group of order 8 vanishes on more than one conjugacy class, we conclude that $|C \cup D \cup E| > \frac{3}{4}|G|$. Consequently either $|C|$, $|D|$ or $|E|$ is bigger than $\frac{1}{4}|G|$. It follows that there exist $g \in C \cup D \cup E$ with $|C_G(g)| < 4$.

Since $|C_G(g)| > 2$, we get that $|C_G(g)| = 3$. Then $\langle g \rangle$ is a Sylow subgroup of order 3. A theorem of Feit and Thompson [5] implies either $G \cong PSL(2,7)$ or G has a nilpotent subgroup N such that G/N is isomorphic to either A_3, S_3 or A_5. Since $PSL(2,7)$, A_3 and A_5 are not rational, we get that $G/N = S_3$.

We need to show that $N = 1$. Suppose the contrary, then $N \neq 1$. Now 3 divides $|G/N|$ so that $g \notin N$ and gN is an element of order 3 in G/N. Therefore $3 \leq \left|C_{G/N}(gN)\right| \leq |C_G(g)| = 3$ forcing $\left|C_{G/N}(gN)\right| = |C_G(g)| = 3$. It follows that every irreducible character of G that does not contain N in its kernel must vanish on g. As χ is faithful we get that $\chi(g) = 0$ and so $g \in C$ and $C \subset G - N$. As $\langle g \rangle \in Syl_3(G)$ and g is rational, every element of order three of G is conjugate to g.

We now show that N is a 2-group. Suppose the contrary, and let p be an odd prime divisor of $|N|$. Since N is nilpotent, p divides $|Z(N)|$. Let $v \in Z(N)$ be of order p. Since v is rational we get that $\frac{N_G(v)}{C_G(v)}$ is a cyclic group of order $p-1$. As $N \subseteq C_G(v)$ we have that $\frac{N_G(v)}{C_G(v)} \cong \frac{N_G(v)/N}{C_G(v)/N} \subseteq \frac{G/N}{C_G(v)/N}$. But $G/N = S_3$ and no factor group of S_3 has a cyclic subgroup of order $p-1$ for $p \neq 3$. We conclude that $p = 3$. But $|G|_3 = |G/N|_3 = 3$ so that $(3, |N|) = 1$, a contradiction. Hence N is a 2-group.

If N has a characteristic subgroup M of order two, then $M \lhd G$ so that $M \subseteq Z(G)$ contradicting the fact that $|C_G(g)| = 3$. Thus N has no characteristic subgroup of order two. It particular N is not cyclic and $|Z(N)| > 2$.

Now $|C \cup D \cup E| > \frac{3}{4}|G|$ so $|D \cup E| > \frac{3}{4}|G| - |C| = \frac{5}{12}|G|$. So either $|D|$ or $|E|$ is bigger than $\frac{5}{24}|G|$. Thus there is an element $h \in D \cup E$ with $|C_G(h)| < \frac{24}{5}$ so that $|C_G(h)| \leq 4$. As $|N| = \frac{1}{6}|G|$, we get that $h \in G - N$.

Again $|C_G(h)| > 2$. Also, $|C_G(h)| = 3$ implies that h has order three and so $h \in C$, a contradiction. We conclude that $|C_G(h)| = 4$. Then h

is a 2-element. Let S be a Sylow 2-subgroup of G containing h. Clearly $N \subseteq S$ with $|S : N| = 2$, and as $|N| > 2$ we have that $|S| \geq 8$. By [8] (p. 375, Satz 14.23) S has maximal class and by [8] (p. 339, Satz 11.9b), we get that S is either Dihedral, generalized Quaternion or Quasidihedral group.

Suppose that $|S| > 8$, then $|N| \geq 8$. Now, N is maximal in S so [7] (Theorem 4.3, p. 191) implies that either N is cyclic or $|Z(N)| = 2$, a contradiction. Therefore $|S| = 8$ and so $|G| = 24$. As $|G : G'| = 2$ we get that $G \cong S_4$ (see, e.g., [11], p. 304). Finally S_4 does not satisfy our assumption, a final contradiction. □

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