Title: The extensibility of $D(-1)$-triples $\{1, b, c\}$

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Let $b = 5, 10, 17, 26, 37$ or 50. In this paper, we show that for those integers c greater than 1 such that both $c-1$ and $bc-1$ are squares, the system of simultaneous Pell equations

$$z^2 - cx^2 = c - 1, \quad bz^2 - cy^2 = c - b$$

has only the trivial solutions $(x, y, z) = (0, \pm\sqrt{b-1}, \pm\sqrt{c-1})$. This implies that there do not exist integers $c, d\,(>1)$ such that the product of any two distinct elements of the set $\{1, b, c, d\}$ diminished by 1 is a square. We also show that in case b is a positive integer with $\sqrt{b-1}$ a prime, if it is true for the smallest eight c's with $c > 1$ for which the set $\{1, b, c\}$ has the property above, then the same is true for all such c's.

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