

Title: On the diophantine equation $x^2 + 2^\alpha 3^\beta 5^\gamma 7^\delta = y^n$

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Let $S = \{p_1, \dots, p_s\}$ be a set of distinct primes and denote by \mathbf{S} the set of non-zero integers composed only of primes from S . Further, denote by Q the product of the primes from S . Let $f \in \mathbb{Z}[X]$ be a monic quadratic polynomial with negative discriminant D_f contained in \mathbf{S} . Consider equation $f(x) = y^n$ (2) in integer unknowns x, y, n with $n \geq 3$ prime and $y > 1$. It follows from a general result of [?] that in (2) n can be bounded from above by an effectively computable constant depending only on Q . This bound is, however, large and is not given explicitly. Using some results of BUGEAUD and SHOREY [?] we derive, apart from certain exceptions, a good and completely explicit upper bound for n in (2) (see Theorems 1 and 2). Further, combining our Theorem 2 with some deep results of COHN [?] and DE WEGER [?] we give all non-exceptional (see Section 1) solutions of equation $x^2 + 2^\alpha 3^\beta 5^\gamma 7^\delta = y^n$ (6), where $x, y, n, \alpha, \beta, \gamma, \delta$ are unknown non-negative integers with $x \geq 1$, $\gcd(x, y) = 1$ and $n \geq 3$ (cf. Theorem 3). When, in (6), $\alpha \geq 1$ is also assumed then our Theorem 3 is a generalization of a result of LUCA [?]. In this case all the solutions of equation (6) are listed.

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