Publ. Math. Debrecen
71/1-2 (2007), 11–20

Derivations with annihilator conditions in prime rings

By BASUDEB DHARA (Kharagpur) and R. K. SHARMA (New Delhi)

Abstract. Let R be a prime ring of char $R \neq 2$ with a derivation d and U a noncentral Lie ideal. If $a \in R$, such that $au^s(d(u))^n u^t \in Z(R)$ for all $u \in U$ and $s(\geq 0)$, $t(\geq 0), n(\geq 1)$ fixed positive integers, then either $a = 0$ or R satisfies S_4, the standard identity in four variables.

1. Introduction

Throughout this paper R always denotes a prime ring with center $Z = Z(R)$, extended centroid C and Q its two-sided Martindale quotient ring. The Lie commutator of x, y is denoted by $[x, y]$ and defined by $[x, y] = xy - yx$ for $x, y \in R$.

In [9], Herstein proved that if $d \neq 0$ is a derivation of a prime ring R such that $(d(x))^n \in Z$ for all $x \in R$, then R satisfies S_4, the standard identity in 4 variables. In [1], Bergen and Carini studied the case for a noncentral Lie ideal. They proved that if R is a prime ring of characteristic not 2 and if d is a nonzero derivation of R satisfying $(d(u))^n \in Z$ for all u in some noncentral Lie ideal of R, then also the same conclusion holds.

Other papers have studied derivations with annihilator conditions. Posner [16] proved that if R is a prime ring and $a \in R$ such that $a\,d(x) = 0$ for all $x \in R$ or $d(x)a = 0$ for all $x \in R$ then either $a = 0$ or $d = 0$. In [3], Brešar proved that if R is a semiprime $(n-1)!$ torsion free ring and if $a\,d(x)^n = 0$ for all $x \in R$, and $a \in R$, n a fixed positive integer then $ad(R) = 0$. In particular, if R is prime then $a = 0$ or $d = 0$. This result was generalized by Lee and Lin [14] for the Lie ideal

Mathematics Subject Classification: 16W25, 16R50, 16N60.
Key words and phrases: prime ring, derivation, extended centroid, Martindale quotient ring.

case without considering R to be $(n-1)!$ torsion free. Lee and Lin's result for prime ring case is as follows:

Let R be a prime ring with a derivation d and let U be a Lie ideal of R, $a \in R$. Suppose that $ad(u)^n = 0$ for all $u \in U$, where n is a fixed integer. Then $ad(U) = 0$ unless char $R = 2$ and $\dim_C RC = 4$. In addition if $[U, U] \neq 0$, then $ad(R) = 0$.

For one-sided ideals, Chang and Lin [4] proved the following:

Let R be a prime ring, ρ a nonzero right ideal of R, d a derivation of R and n a fixed positive integer. If $d(u)u^n = 0$ for all $u \in \rho$, then $d(\rho)\rho = 0$ and if $u^n d(u) = 0$ for all $u \in \rho$, then $d = 0$ unless $R \cong M_2(F)$, the 2×2 matrices over a field F of two elements.

Recently we obtained results [17] for a prime ring R with a derivation d and U a nonzero Lie ideal that if $a \in R$ such that $a(d(u))^n u^m = 0$ for all $u \in U$ or $au^m(d(u))^n = 0$ for all $u \in U$, m, n are fixed positive integers, then (i) $a = 0$ or $d(U) = 0$ if char $R \neq 2$ and (ii) $a = 0$ or $d(R) = 0$ if $[U, U] \neq 0$ and $R \ncong M_2(F)$.

Here we generalize most of the above results by considering the cases $au^s(d(u))^n u^t = 0$ for all $u \in U$ and $au^s(d(u))^n u^t \in Z(R)$ for all $u \in U$, a nonzero Lie ideal of R.

One can find a nonzero derivation d, a nonzero Lie ideal U of R, and a nonzero $a \in R$ such that $au^s(d(u))^n u^t \in Z(R)$ for all $u \in U$ and for suitable nonnegative integers s, n, t.

Example. Let $R = M_2(F)$, the ring of all 2×2 matrices over the field F. Take $U = R$ as a non-central Lie ideal of R and $d(x) = [q, x]$ as a nonzero inner derivation induced by some $q \in R$. Then, since $[x, y]^2 \in Z(R)$ for all $x, y \in R$, we have for any $0 \neq a \in Z(R)$ and $s = t = 0$, $n = 2$ that $au^s(d(u))^n u^t \in Z(R)$ for all $u \in U$.

2. Main results

First we prove a lemma

Lemma 2.1. *Let $R = M_2(F)$, the ring of 2×2 matrices over a field F of characteristic $\neq 2$. If for some $a, b \in R$, $a[x, y]^s[b, [x, y]]^n[x, y]^t = 0$ for all $x, y \in R$, where $s(\geq 0)$, $t(\geq 0)$, $n(\geq 1)$ are fixed integers, then either $a = 0$ or $b \in F \cdot I_2$.*

PROOF. Let $a = (a_{ij})_{2\times 2}$ and $b = (b_{ij})_{2\times 2}$. We choose $x = e_{12}$, $y = e_{21}$. Then the identity $a[x,y]^s[b,[x,y]]^n[x,y]^t = 0$ gives

$$0 = \begin{cases} (-1)^{n/2}2^n(b_{12}b_{21})^{n/2}\begin{pmatrix} a_{11} & (-1)^{s+t}a_{12} \\ a_{21} & (-1)^{s+t}a_{22}\end{pmatrix}, & \text{if } n \text{ is even} \\ (-1)^{(n-1)/2}2^n(b_{12}b_{21})^{(n-1)/2}\begin{pmatrix} (-1)^s a_{12}b_{21} & (-1)^{t+1}a_{11}b_{12} \\ (-1)^s a_{22}b_{21} & (-1)^{t+1}a_{21}b_{12}\end{pmatrix}, & \text{if } n \text{ is odd.}\end{cases}$$

This implies that if $b_{12} \neq 0$, $b_{21} \neq 0$ then $a = 0$.

Let $a \neq 0$. Then at least one of b_{12} and b_{21} must be zero. So without loss of generality we assume that $b_{12} = 0$. Then assuming $x = e_{11}$, $y = e_{12} - e_{21}$ we get

$$[b,[x,y]]^n = \begin{cases} \lambda^{n/2}I, & \text{if } n \text{ is even} \\ \lambda^{(n-1)/2}\begin{pmatrix} -b_{21} & b_{11}-b_{22} \\ -(b_{11}-b_{22}) & b_{21}\end{pmatrix}, & \text{if } n \text{ is odd}\end{cases}$$

where $\lambda = b_{21}^2 - (b_{11} - b_{22})^2$.
If n is even then the identity $a[x,y]^s[b,[x,y]]^n[x,y]^t = 0$ gives

$$0 = \begin{cases} \lambda^{n/2}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}, & \text{if } s+t \text{ is even} \\ \lambda^{n/2}\begin{pmatrix} a_{12} & a_{11} \\ a_{22} & a_{21}\end{pmatrix}, & \text{if } s+t \text{ is odd.}\end{cases}$$

which implies that $\lambda = 0$, since $a \neq 0$.
If n is odd then we have

$$[x,y]^s[b,[x,y]]^n[x,y]^t = \begin{cases} (-1)^s\lambda^{(n-1)/2}\begin{pmatrix} -b_{21} & b_{11}-b_{22} \\ -(b_{11}-b_{22}) & b_{21}\end{pmatrix}, & \text{if } s+t \text{ is even} \\ (-1)^s\lambda^{(n-1)/2}\begin{pmatrix} b_{11}-b_{22} & -b_{21} \\ b_{21} & -(b_{11}-b_{22})\end{pmatrix}, & \text{if } s+t \text{ is odd.}\end{cases}$$

If n is odd and $s+t$ is even then the identity $a[x,y]^s[b,[x,y]]^n[x,y]^t = 0$ becomes

$$(-1)^s\lambda^{(n-1)/2}\begin{pmatrix} -a_{11}b_{21} - a_{12}(b_{11}-b_{22}) & a_{11}(b_{11}-b_{22}) + a_{12}b_{21} \\ -a_{21}b_{21} - a_{22}(b_{11}-b_{22}) & a_{21}(b_{11}-b_{22}) + a_{22}b_{21}\end{pmatrix} = 0.$$

If $\lambda \neq 0$, then this implies that

$$\begin{aligned} -a_{11}b_{21} - a_{12}(b_{11} - b_{22}) &= 0, \\ a_{11}(b_{11} - b_{22}) + a_{12}b_{21} &= 0, \\ -a_{21}b_{21} - a_{22}(b_{11} - b_{22}) &= 0, \\ a_{21}(b_{11} - b_{22}) + a_{22}b_{21} &= 0. \end{aligned}$$

From these equations we get

$$a_{11}\lambda = 0, \qquad a_{22}\lambda = 0,$$

$$a_{12}\lambda = 0, \qquad a_{21}\lambda = 0.$$

Since $\lambda \neq 0$, $a = 0$, a contradiction.

Thus $\lambda = 0$. Similarly, if n is odd and $s + t$ is also odd then it can be proved that $\lambda = 0$.

On the other hand, by choosing $x = e_{11}$, $y = e_{12} + e_{21}$ we obtain in a similar manner that

$$\mu = b_{21}^2 + (b_{11} - b_{22})^2 = 0.$$

Hence $0 = \lambda \pm \mu$ leads $b_{21} = 0$ and $b_{11} = b_{22}$. So b is scalar. Thus we have proved that either $a = 0$ or $b \in F \cdot I_2$. □

Before proving the main theorem, we introduce some remarks.

Remark 1. Denote by $T = Q *_C C\{X\}$, the free product over C of the C-algebra Q and the free C-algebra $C\{X\}$, with X the countable set consisting of the noncommuting indeterminates $x_1, x_2, \ldots$.

Elements of T are called generalized polynomials. Nontrivial generalized polynomial means a nonzero element of T. Any element $m \in T$ of the form $m = q_0y_1q_1y_2q_2 \ldots y_nq_n$, where $\{q_0, q_1, \ldots, q_n\} \subseteq Q$ and $\{y_1, y_2, \ldots, y_n\} \subseteq X$, is called a monomial and $q_0, q_1, \ldots, q_n$ are called the coefficients of m. Each $f \in T$ can be represented as a finite sum of monomials, and such representation is not unique. Let B be a set of C-independent vectors of Q. A B-monomial is a monomial of the form $q_0y_1q_1y_2q_2 \ldots y_nq_n$, where $\{q_0, q_1, \ldots, q_n\} \subseteq B$ and $\{y_1, y_2, \ldots, y_n\} \subseteq X$. Let $V = BC$, the C-subspace spanned by B. Then f is called a V-generalized polynomial if and only if f has a presentation with all of its coefficients in V. Thus any V-generalized polynomial f can be written in the form $f = \sum \alpha_i m_i$, where $\alpha_i \in C$ and m_i are B-monomials and this representation is unique. This V-generalized polynomial $f = \sum \alpha_i m_i$ is trivial i.e., zero element in T if and only if $\alpha_i = 0$ for each i. For detail study we refer to [5].

This simple criterion will be used in the proof of the theorem to assure that R satisfies a nontrivial generalized polynomial identity.

Remark 2. It is well known that if U is a noncommutative Lie ideal of a prime ring R and I is the ideal of R generated by $[U,U]$, then $I \subseteq U + U^2$ and $[I,I] \subseteq U$ (see [12, Lemma 2 (i),(ii)]).

Briefly we give its proof. For $a, b \in U$ and $r \in R$, we have $[a,b]r = [ar,b] - a[r,b] \in U + U^2$. For $s \in R$, we get commuting both sides by s that $s[a,b]r = [a,b]rs + [[ar,b],s] - [a[r,b],s] \in U + U^2$, since $[a[r,b],s] = a[[r,b],s] + [a,s][r,b] \in U^2$. Thus $I \subseteq U + U^2$. Now since $[U^2, I] \subseteq U$ holds true by using the identity $[xy,z] = [x,yz] + [y,zx]$ for $x, y \in U$ and $z \in I$, we have $[I,I] \subseteq U$.

We are now in a position to prove our theorem

Theorem 2.2. *Let R be a prime ring with a derivation d and U be a nonzero Lie ideal. If $a \in R$, such that $au^s(d(u))^n u^t = 0$ for all $u \in U$ and $s(\geq 0)$, $t(\geq 0)$, $n(\geq 1)$ fixed integers, then*

(i) *$a = 0$ or $d(U) = 0$ if U is central,*

(ii) *$a = 0$ or $d(R) = 0$ if char $R \neq 2$ and U is noncentral,*

(iii) *$a = 0$ or $d(R) = 0$ or char $R = 2$ and R satisfies S_4 if U is noncommutative.*

Proof. (i) If U is central i.e., $U \subseteq Z$ then $d(U) \subseteq Z$, as $d(Z) \subseteq Z$. Since the center of a prime ring R contains no zero divisor of R, $au^s(d(u))^n u^t = 0$ implies that either $a = 0$ or $d(u) = 0$.

(ii) Now assume that char $R \neq 2$ and U is noncentral. Since char $R \neq 2$, by [2, Lemma 1] $[U,U] \neq 0$ and $0 \neq [I,R] \subseteq U$, where I is the ideal generated by $[U,U]$. So $[I,I] \subseteq U$. Hence without loss of generality we can assume $U = [I,I]$. By our assumption we have,

$$a[x,y]^s(d([x,y]))^n[x,y]^t = 0 \tag{1}$$

for all $x, y \in I$, which implies

$$a[x,y]^s([d(x),y] + [x,d(y)])^n[x,y]^t = 0$$

for all $x, y \in I$. If d is not Q-inner then by Kharchenko's theorem [11],

$$a[x,y]^s([u,y] + [x,v])^n[x,y]^t = 0$$

for all $x, y, u, v \in I$.

By Chuang [5, Theorem 2], this generalized polynomial identity (GPI) is also satisfied by Q and hence by R. In particular for $v = 0, u = x$, we get

$$a[x, y]^{s+n+t} = 0 \tag{2}$$

for all $x, y \in R$. Let $w = [x, y]^{s+n+t}$. Then $aw = 0$. From (2) we can write $a[p, wqa]^{s+n+t} = 0$ for all $p, q \in R$. Since $aw = 0$, it reduces to $a(pwqa)^{s+n+t} = 0$. This can be written as $(wqap)^{s+n+t+1} = 0$ for all $p, q \in R$. By Levitzki's lemma [7, Lemma 1.1], $wqa = 0$ for all $q \in R$. Since R is prime, either $a = 0$ or $w = 0$. If $a \neq 0$ then $w = [x, y]^{s+n+t} = 0$ for all $x, y \in R$. Then by Herstein [8, Theorem 2], R is commutative, contradicting the fact that $0 \neq U$ is noncentral. Now if d is Q-inner i.e., $d(x) = [b, x]$ for all $x \in R$ and for some $b \in Q$, then (1) becomes

$$a[x, y]^s[b, [x, y]]^n[x, y]^t = 0$$

for all $x, y \in I$. By Chuang [5, Theorem 2], this GPI is also satisfied by Q i.e.,

$$f(x, y) = a[x, y]^s[b, [x, y]]^n[x, y]^t = 0 \tag{3}$$

for all $x, y \in Q$.

In case the center C of Q is infinite, we have $f(x, y) = 0$ for all $x, y \in Q \otimes_C \overline{C}$, where $\overline{C}$ is the algebraic closure of C. Since both Q and $Q \otimes_C \overline{C}$ are prime and centrally closed [6, Theorem 2.5 and 3.5], we may replace R by Q or $Q \otimes_C \overline{C}$ according to C finite or infinite. Thus we may assume that R is centrally closed over C (i.e., $RC = R$) which is either finite or algebraically closed and $f(x, y) = 0$ for all $x, y \in R$.

Now consider two cases.

Case I. R satisfies a nontrivial GPI

By Martindale's theorem [15], R is then a primitive ring having nonzero socle H with C as the associated division ring. Hence by Jacobson's theorem [10, p.75] R is isomorphic to a dense ring of linear transformations of some vector space V over C, and H consists of the linear transformations in R of finite rank. If V is a finite dimensional over C then the density of R on V implies that $R \cong M_k(C)$ where $k = \dim_C V$.

Suppose that $\dim_C V \geq 3$.

We show that for any $v \in V$, v and bv are linearly C-dependent. Suppose that v and bv are linearly independent for some $v \in V$. Since $\dim_C V \geq 3$, there exists $w \in V$ such that v, bv, w are linearly independent over C. By density there exist $x, y \in R$ such that

$$xv = 0, \qquad xbv = v, \qquad xw = v + 2bv$$

$$yv = bv, \qquad ybv = w, \qquad yw = 0.$$

Then $[x, y]v = (xy - yx)v = v$, $[x, y]bv = (xy - yx)bv = xw - yv = v + bv$ and so $[b, [x, y]]^n v = (-1)^n v$. Hence

$$0 = a[x, y]^s [b, [x, y]]^n [x, y]^t v = (-1)^n av.$$

This implies that if $av \neq 0$, then v and bv are linearly C-dependent. Now suppose that $av = 0$. Since $a = 0$ finishes the proof of the theorem, we assume $a \neq 0$. Since $a \neq 0$, there exists $w \in V$ such that $aw \neq 0$ and then $a(v + w) = aw \neq 0$. By the previous argument we have that w, bw are linearly C-dependent and $(v + w)$, $b(v + w)$ are also. Thus there exist $\alpha, \beta \in C$ such that $bw = w\alpha$ and $b(v + w) = (v + w)\beta$. Moreover, v and w are clearly C-independent and so by density there exist $x, y \in R$ such that

$$xw = 0, \qquad xv = v + w$$

$$yw = v + w, \qquad yv = v.$$

Then we obtain by using $av = 0$ that

$$0 = a[x, y]^s [b, [x, y]]^n [x, y]^t w = \pm aw(\beta - \alpha)^n.$$

Since $aw \neq 0$, $\alpha = \beta$ and so $bv = v\alpha$ contradicting the independency of v and bv. Hence for each $v \in V$, $bv = v\alpha_v$ for some $\alpha_v \in C$. It is very easy to prove that α_v is independent of the choice of $v \in V$. Thus we can write $bv = v\alpha$ for all $v \in V$ and $\alpha \in C$ fixed.

Now let $r \in R$, $v \in V$. Since $bv = v\alpha$,

$$[b, r]v = (br)v - (rb)v = b(rv) - r(bv) = (rv)\alpha - r(v\alpha) = 0.$$

Thus $[b, r]v = 0$ for all $v \in V$ i.e., $[b, r]V = 0$. Since $[b, r]$ acts faithfully as a linear transformation on the vector space V, $[b, r] = 0$ for all $r \in R$. Therefore $b \in Z(R)$ implies $d = 0$, ending the proof of this part.

Now suppose $\dim_C V = 2$. Then $R \cong M_2(C)$. Since char $R \neq 2$, by Lemma 2.1 we have that either $a = 0$ or $b \in C \cdot I_2$. Now $b \in C \cdot I_2$ implies $d = 0$.

Case II. R does not satisfy any nontrivial GPI
Assume that $a \neq 0$ and $d \neq 0$. Since $d \neq 0$, $b \notin C$. Let $T = Q *_C C\{x, y\}$, the free product of C-algebra Q and $C\{x, y\}$, the free C-algebra in noncommuting indeterminates x and y. By assumption $a[x, y]^s(b[x, y] - [x, y]b)^n[x, y]^t$ is a GPI for R and so

$$f(x, y) = a[x, y]^s(b[x, y] - [x, y]b)^n[x, y]^t = 0$$

in T, since R has no nonzero GPI. Expansion of it yields that if the coefficients $\{1, b, b^2\}$ are C-independent, then all the monomials in the expansion are basis monomials in T and thus $f(x, y) \neq 0$ in T, a contradiction. On the other hand if $b^2 \in span_C\{1, b\}$, it is true that the basis monomial $a[x, y]^s(b[x, y])^n[x, y]^t$ is not canceled in the expansion, so again $f(x, y) \neq 0$ in T, a contradiction.

Thus either $a = 0$ or $d = 0$.

(iii) Since U is noncommutative, by [12, Lemma 2], $[M, M] \subseteq U$ where M is the ideal generated by $[U, U]$. By the similar argument in the proof of part (ii) we have either $a = 0$ or $d = 0$ or char $R = 2$ and $R \subseteq M_2(F)$ for some field F i.e., either $a = 0$ or $d = 0$ or char $R = 2$ and R satisfies S_4. This completes the proof of this part. □

Theorem 2.3. *Let R be a prime ring of char $R \neq 2$ with a nonzero derivation d and U be a noncentral Lie ideal. If $a \in R$, such that $au^s(d(u))^n u^t \in Z(R)$ for all $u \in U$ and $s(\geq 0)$, $t(\geq 0)$, $n(\geq 1)$ fixed integers, then either $a = 0$ or R satisfies S_4, the standard identity in four variables.*

Proof. Assume that $a \neq 0$. Since char $R \neq 2$ and U is noncentral, by [2, Lemma 1], there exists an ideal I of R such that $0 \neq [I, R] \subseteq U$ and $[U, U] \neq 0$. Let J be any nonzero two-sided ideal of R. Then it is easy to check that $V = [I, J^2] \subseteq U$ is a noncentral Lie ideal of R. If for each $v \in V$, $av^s(d(v))^n v^t = 0$, then by Theorem 2.2, $d = 0$ which contradicts our assumption. Hence for some $v \in V$, $0 \neq av^s(d(v))^n v^t \in J \cap Z(R)$, since $d(V) \subseteq J$. Thus $J \cap Z(R) \neq 0$. Now let K be a nonzero two-sided ideal of R_Z, the ring of central quotients of R. Since $K \cap R$ is a nonzero two-sided ideal of R, $(K \cap R) \cap Z(R) \neq 0$. Therefore, K contains an invertible element in R_Z and so R_Z is a simple ring with identity 1.

Moreover, without loss of generality, we may assume that $U = [I, I]$. Thus I satisfies the generalized differential identity

$$[a[x_1, x_2]^s(d[x_1, x_2])^n[x_1, x_2]^t, x_3]. \tag{4}$$

If d is not Q-inner then by Kharchenko's theorem [11],

$$[a[x_1, x_2]^s([y_1, x_2] + [x_1, y_2])^n[x_1, x_2]^t, x_3] = 0 \tag{5}$$

for all $x_1, x_2, x_3, y_1, y_2 \in I$. By CHUANG [5], this GPI is also satisfied by Q and hence by R. By localizing R at $Z(R)$, it follows that $[a[x_1, x_2]^s([y_1, x_2] + [x_1, y_2])^n[x_1, x_2]^t, x_3]$ is also an identity of R_Z. Since R and R_Z satisfy the same polynomial identities, in order to prove that R satisfies S_4, we may assume that R is simple ring with 1 and $[R, R] \subseteq U$. Thus R satisfies the identity (5). Now putting $y_1 = [b, x_1] = \delta(x_1)$ and $y_2 = [b, x_2] = \delta(x_2)$ for some $b \notin Z(R)$, where δ is an inner derivation induced by some $b \in R$, we obtain that R satisfies

$$[a[x_1, x_2]^s([y_1, x_2] + [x_1, y_2])^n[x_1, x_2]^t, x_3] = 0.$$

Thus by MARTINDALE's theorem [15], R is a primitive ring with minimal right ideal, whose commuting ring D is a division ring which is finite dimensional over $Z(R)$. However, since R is simple with 1, R must be Artinian. Hence $R = D_{k'}$, the $k' \times k'$ matrices over D, for some $k' \geq 1$. Again by [13, Lemma 2], it follows that there exists a field F such that $R \subseteq M_k(F)$, the ring of $k \times k$ matrices over the field F, and $M_k(F)$ satisfies

$$[a[x_1, x_2]^s(\delta[x_1, x_2])^n[x_1, x_2]^t, x_3] = 0.$$

If $k \geq 3$, then by substituting $x_1 = e_{12},\ x_2 = e_{22}$ we see that the rank of $[x_1, x_2]$ is equal to 1 and thus the rank of $a[x_1, x_2]^s(\delta[x_1, x_2])^n[x_1, x_2]^t$ is ≤ 2. Therefore $a[x_1, x_2]^s(\delta[x_1, x_2])^n[x_1, x_2]^t = 0$ for all $x_1, x_2 \in M_k(F)$. Since char $F \neq 2$, by Theorem 2.2, we get either $a = 0$ or $\delta = 0$ i.e., $b \in Z(R)$. In both cases we have a contradiction. Thus $k = 2$, that is, R satisfies S_4. □

Similar arguments can be adapted to draw the same conclusion in case d is a Q-inner derivation induced by some $b \in Q$.

ACKNOWLEDGMENT. The authors wish to thank the referees for their valuable comments and suggestions for the improvement of this paper.

References

[1] J. BERGEN and L. CARINI, A note on derivations with power central values on a Lie ideal, *Pac. J. Math.* **132**(2) (988), 209–213.

[2] J. BERGEN, I. N. HERSTEIN and J. W. KEER, Lie ideals and derivations of prime rings, *J. Algebra* **71** (1981), 259–267.

[3] M. BREŠAR, A note on derivations, *Math. J. Okayama Univ.* **32** (1990), 83–88.

[4] CHI-MING CHANG and YU-CHING LIN, Derivations on one-sided ideals of prime rings, *Tamsui Oxford J. Math. Sci.* **17**(2) (2001), 139–145.

[5] C. L. Chuang, GPI's having coefficients in Utumi quotient rings, *Proc. Amer. Math. Soc.* **103**(3) (1988), 723–728.

[6] T. S. Erickson, W. S. Martindale III and J. M. Osborn, Prime nonassociative algebras, *Pacific J. Math.* **60** (1975), 49–63.

[7] I. N. Herstein, Topics in ring theory, *Univ. of Chicago Press*, *Chicago*, 1969.

[8] I. N. Hesrein, Center-like elements in prime rings, *J. Algebra* **60** (1979), 567–574.

[9] I. N. Herstein, Derivations of prime rings having power central values, Algebraist's Homage, Contemporary Mathematics Vol. 13, *A.M.S., Providence, Rhode Island*, 1982.

[10] N. Jacobson, Structure of rings, *Amer. Math. Soc. Colloq. Pub.* **37**, Amer. Math. Soc., Providence, RI, 1964.

[11] V. K. Kharchenko, Differential identity of prime rings, *Algebra and Logic.* **17** (1978), 155–168.

[12] C. Lanski, Differential identities, Lie ideals, and Posner's theorems, *Pacific J. Math.* **134**(2) (1988), 275–297.

[13] C. Lanski, An engel condition with derivation, *Proc. Amer. Math. Soc.* **118**(3) (1993), 731–734.

[14] T. K. Lee and J. S. Lin, A result on derivations, *Proc. Amer. Math. Soc.* **124**(6) (1996), 1687–1691.

[15] W. S. Martindale III, Prime rings satisfying a generalized polynomial identity, *J. Algebra* **12** (1969), 576–584.

[16] E. C. Posner, Derivation in prime rings, *Proc. Amer. Math. Soc.* **8** (1957), 1093–1100.

[17] R. K. Sharma and B. Dhara, An annihilator condition on prime rings with derivations, *Tamsui Oxf. J. Math. Sci.* **21**(1) (2005), 71–80.

BASUDEB DHARA
DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
KHARAGPUR-721302
INDIA

E-mail: basu_dhara@yahoo.com

R. K. SHARMA
DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
HAUZ KHAS, NEW DELHI 110016
INDIA

E-mail: rksharma@maths.iitd.ac.in

(Received July 13, 2005; revised May 17, 2006)