Title: On a parameterized family of relative Thue equations

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Let $k := \mathbb{Q}(\sqrt{-D})$ be an imaginary quadratic number field and $\mathbb{Z}_k$ be the corresponding ring of integers. We consider the family of relative Thue equations

$$F_t(x, y) = x^3 - (t-1)x^2 y - (t+2)xy^2 - y^3 = \ell$$

with $t, \ell \in \mathbb{Z}_k, t \notin \mathbb{Z}$ and $|\ell| \leq |2t + 1|$. Let $k(\alpha)$ be the cubic extension of k generated by a root α of the polynomial $f_t(x) = F_t(x, 1)$, and let $\mathbb{Z}_{k(\alpha)}$ be its ring of integers. A pair (x, y) with $x, y \in \mathbb{Z}_k$ is a solution of the Thue equation if and only if the element $\gamma = x - \alpha y \in \mathbb{Z}_{k(\alpha)}$ has a norm satisfying $|N_{k(\alpha)/k}(\gamma)| \leq |2t + 1|$. We determine all elements of $\mathbb{Z}_{k(\alpha)}$ having norms less than or equal to $|2t + 1|$. Further we solve the above Thue equation for all $t \in \mathbb{Z}_k, t \notin \mathbb{Z}$ with $\Re t = -\frac{1}{2}$ and all $|\ell| \leq |2t + 1|$.

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