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**Title:** On homomorphisms of an abelian group into the group of invertible formal power series

**Author(s):** Wojciech Jabłoński and Ludwig Reich

We study solutions of the translation equation in rings of formal power series  $\mathbb{K}[[X]]$ ,  $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$  (the so called one-parameter groups or flows), and even, more generally, homomorphisms  $\Theta$  from an abelian group  $(G, +)$  into the group  $(\Gamma, \circ)$  of invertible power series in  $\mathbb{K}[[X]]$ . This problem can equivalently be formulated as the question of finding homomorphisms  $\Phi$  from  $(G, +)$  into the differential group  $L_\infty^1 = (Z_\infty, \cdot)$  describing the chain rules of higher order of  $C^\infty$ -functions with fixed point 0. We prove the general form of the homomorphisms  $\Theta : G \rightarrow \Gamma$ ,  $\Theta(t) = \sum_{k=1}^{\infty} c_k(t)X^k$  and  $\Phi : G \rightarrow Z_\infty$ ,  $\Phi = (f_n)_{n \geq 1}$ , for which  $c_1$  and  $f_1$  take infinitely many values (Theorems ?? and ??). These representations use sequences  $(P_n)_{n \geq 2}$  of universal polynomials in  $c_1$ , and  $(v_n)_{n \geq 2}$  of universal polynomials in  $f_1$ , and some sequences of parameters, which determine the individual homomorphism. We describe the connection between these forms of the homomorphisms. These results are deduced from the special case  $|f_1| \neq 1$  (Theorem ??) and the case when  $c_1$  is a regular function (Theorem ??).

**Address:**

Wojciech Jabłoński  
Department of Mathematics  
University of Rzeszow  
Rejtana 16 A  
35-310 Rzeszów  
POLAND

**Address:**

Ludwig Reich  
Institute of Mathematics  
Karl-Franzens-University Graz  
Heinrichstrasse 36  
A-8010 Graz  
AUSTRIA