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**Title:** The existence of an associate subgroup in normal cryptogroups

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Let  $S$  be a semigroup. If  $a, x \in S$  are such that  $a = axa$ , then  $x$  is an associate of  $a$ . A subgroup  $G$  of  $S$  is an associate subgroup of  $S$  if it contains exactly one associate of each element of  $S$ . Representing a normal cryptogroup  $S$  as a strong semilattice of Rees matrix semigroups, we give necessary and sufficient conditions on  $S$  in order for  $S$  to have an associate subgroup. Having an associate subgroup is equivalent to admitting a unary operation satisfying three simple axioms. We prove that every maximal subgroup of  $S$  is an associate subgroup if and only if  $S$  is completely simple. A counterexample shows that the unary semigroups corresponding to two different associate subgroups of (completely simple)  $S$  need not be isomorphic. Normal cryptogroups having an associate subgroup are characterized in several ways in the main result of the paper.

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